Compactness on Fuzzy Soft r -Minimal Spaces

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Abstract

In this study, a new form of fuzzy r -minimal structure [28] called a fuzzy soft r -minimal structure, is defined, which is an extension of the fuzzy soft topology introduced by Aygunoglu et al. [26]. In addition, the concepts of fuzzy soft r -minimal continuity and fuzzy soft r -minimal compactness are introduced in fuzzy soft r -minimal spaces. Some interesting properties and characterizations are also discussed. Finally, several types of fuzzy soft r -minimal compactness are defined, and the relationships between them are characterized.

Keywords: Fuzzy soft set, Fuzzy soft topology, Fuzzy soft r -minimal structure, Fuzzy soft r -minimal space, Fuzzy soft r -minimal continuity, Fuzzy soft r -minimal compactness

1. Introduction

In 1999, Molodtsov [1] proposed a completely new concept called a soft set theory to model uncertainty, which associates a set with a set of parameters. Using a soft set, we mean a pair (F, E) , where E is a set interpreted as the set of parameters, and the mapping $F : E \rightarrow P(X)$ is referred to as the soft structure on X . After an introduction of the notion of soft sets, several researchers have improved this concept. Soft set theory has been applied to many different fields with great success. Maji et al. [2] introduced the concept of a fuzzy soft set that combines fuzzy sets [3] with soft sets [1]. Soft set and fuzzy soft set theories have a rich potential for several types of applications. Thus far, many spectacular and creative studies on the theories of soft sets and fuzzy soft sets have been conducted (see [2,4–25]). In addition, Aygunoglu et al. [26] studied the topological structure of fuzzy soft sets based on the approach by Sostak [27].

The concept of a fuzzy r -minimal structure was introduced by Yoo et al. [28], and is an extension of the fuzzy topology introduced by Sostak [27]. In addition, the concepts of fuzzy r -minimal continuity and fuzzy r -minimal compactness were introduced in [28] and [29]. Later, Abbas and Min [30] introduced the concept of a (r, s) -fuzzy minimal structure, which is an extension of the intuitionistic fuzzy topology introduced by Mondal and Samanta [31]. In addition, the concepts of (r, s) -fuzzy minimal continuity and (r, s) -fuzzy minimal compactness were introduced in [30].

The aim of this study is to introduce and study the concepts of a fuzzy soft r -minimal structure, fuzzy soft r -minimal space, fuzzy soft r -minimal continuity, and fuzzy soft r -minimal compactness based on the approach developed by Aygunoglu et al. [26]. In addition, their properties are investigated. Later, several types of fuzzy soft r -minimal compactness are defined, and the relationships among them are discussed.

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2. Preliminaries

Throughout this paper, X refers to an initial universe, E is the set of all parameters for X and $A \subseteq E$, I^X is the set of all fuzzy sets on X (where $I = [0, 1]$, $I_0 = (0, 1]$), and for $\alpha \in I$, $\alpha(x) = \alpha$, for all $x \in X$.

Definition 2.1 [2,4,26]. A fuzzy soft set f_A over X is a mapping from E to I^X such that $f_A(e)$ is a fuzzy set on X for each $e \in A$ and $f_A(e) = \underline{0}$, if $e \notin A$, where $\underline{0}$ is a zero function on X . The fuzzy set $f_A(e)$ for each $e \in E$ is called an element of the fuzzy soft set f_A . Here, (X, E) denotes the collection of all fuzzy soft sets on X and is called a fuzzy soft universe [32].

Definition 2.2 [33]. A fuzzy soft point e_{x_t} over X is a fuzzy soft set defined as follows: $e_{x_t}(k) = x_t$, if $k = e$ and $e_{x_t}(k) = \underline{0}$, if $k \in E - \{e\}$, where x_t is a fuzzy point. A fuzzy soft point e_{x_t} is said to belong to a fuzzy soft set f_A , denoted by $e_{x_t} \tilde{\in} f_A$ if $t < f_A(e)(x)$.

Definition 2.3 [26]. A mapping $\tau : E \rightarrow [0, 1]^{(X, E)}$ is called a fuzzy soft topology on X if it satisfies the following conditions for each $e \in E$.

1. $\tau_e(\Phi) = \tau_e(\tilde{E}) = 1$.
2. $\tau_e(f_A \sqcap g_B) \geq \tau_e(f_A) \wedge \tau_e(g_B)$, $\forall f_A, g_B \in (X, E)$.
3. $\tau_e(\bigsqcup_{i \in \Delta} (f_A)_i) \geq \bigwedge_{i \in \Delta} \tau_e((f_A)_i)$, $\forall (f_A)_i \in (X, E)$, $i \in \Delta$.

Then, the pair (X, τ_E) is called a fuzzy soft topological space (FSTS).

All definitions and properties of fuzzy soft sets and fuzzy soft topologies are found in [2,4,12,16,24,26].

3. r -Minimal Structures and r -Minimal Spaces through Fuzzy Soft Sets

In this section, a new form of a fuzzy r -minimal structure called a fuzzy soft r -minimal structure is defined, which is an extension of the fuzzy soft topology introduced by Aygunoglu et al. [26]. In addition, the concepts of fuzzy soft r -minimal continuity and fuzzy soft r -minimal compactness are introduced in fuzzy soft r -minimal spaces. Some interesting properties and characterizations are discussed.

Definition 3.1. Let X be a nonempty set and $r \in I_0$. Fuzzy soft mapping $\widetilde{M} : E \rightarrow [0, 1]^{(X, E)}$ on X is said to be a fuzzy soft r -minimal structure if the family $\widetilde{M}_{e,r} = \{f_A \in (X, E) \mid \widetilde{M}_e(f_A) \geq r\}$ for each $e \in E$ contains Φ and $\tilde{E}$.

Then, $(X, \widetilde{M})$ is called a fuzzy soft r -minimal space (simply, r -FMS). Every member of $\widetilde{M}_{e,r}$ is called a fuzzy soft r -minimal open set. A fuzzy soft set f_A is called a fuzzy soft r -minimal closed set if the complement of f_A is a fuzzy soft r -minimal open set.

Example 3.1. Let (X, τ_E) be an FSTS. It is then easy to see that for each $e \in E$ and $r \in I_0$, a fuzzy soft mapping $\widetilde{M}$ defined by $\widetilde{M}_{e,r} = \{f_A \in (X, E) \mid \tau_e(f_A) \geq r\}$ is a fuzzy soft r -minimal structure on X .

Definition 3.2. [12] Let (X, τ_E) be an FSTS. For $f_A \in (X, E)$ and $r \in I_0$, f_A is called r -fuzzy soft regularly open if $f_A = I_\tau(e, C_\tau(e, f_A, r), r)$.

Definition 3.3. Let (X, τ_E) be an FSTS. For $f_A \in (X, E)$ and $r \in I_0$.

- (1) f_A is called r -fuzzy soft preopen if $f_A \sqsubseteq I_\tau(e, C_\tau(e, f_A, r), r)$.
- (2) f_A is called r -fuzzy soft β -open if $f_A \sqsubseteq C_\tau(e, I_\tau(e, C_\tau(e, f_A, r), r), r)$.

The following implications hold:

r -fuzzy soft regularly open $\Rightarrow$ r -fuzzy soft preopen $\Rightarrow$ r -fuzzy soft β -open.

In general, the converses are not true.

Example 3.2. Let $X = \{x, y\}$ be a classical set and $E = \{e_1, e_2\}$ be the parameter set of X . Define f_E and $g_E \in (X, E)$ as follows: $f_E = \{(e_1, \{\frac{x}{0.3}, \frac{y}{0.4}\}), (e_2, \{\frac{x}{0.3}, \frac{y}{0.4}\})\}$, $g_E = \{(e_1, \{\frac{x}{0.6}, \frac{y}{0.2}\}), (e_2, \{\frac{x}{0.6}, \frac{y}{0.2}\})\}$. The fuzzy soft topology $\tau_E : E \rightarrow [0, 1]^{(X, E)}$ is as follows:

$$\tau_{e_1}(w_E) = \begin{cases} 1, & \text{if } w_E \in \{\Phi, \tilde{E}\}, \\ \frac{1}{4}, & \text{if } w_E = f_E, \\ \frac{1}{3}, & \text{if } w_E = g_E, \\ \frac{1}{3}, & \text{if } w_E = f_E \sqcap g_E, \\ \frac{1}{4}, & \text{if } w_E = f_E \sqcup g_E, \\ 0, & \text{otherwise,} \end{cases}$$

$$\tau_{e_2}(w_E) = \begin{cases} 1, & \text{if } w_E \in \{\Phi, \tilde{E}\}, \\ \frac{1}{4}, & \text{if } w_E = f_E, \\ \frac{1}{2}, & \text{if } w_E = g_E, \\ \frac{1}{2}, & \text{if } w_E = f_E \sqcap g_E, \\ \frac{1}{4}, & \text{if } w_E = f_E \sqcup g_E, \\ 0, & \text{otherwise.} \end{cases}$$

Then, g_E is a $\frac{1}{4}$ -fuzzy soft preopen set and $\frac{1}{4}$ -fuzzy soft

β -open set but is not a $\frac{1}{4}$ -fuzzy soft regularly open set.

Remark 1. Let (X, τ_E) be an FSTS and $r \in I_0$. We denote the set of all r -fuzzy soft regularly open (respectively, r -fuzzy soft preopen and r -fuzzy soft β -open) sets by $r\text{-}FRO(X)$ (respectively, $r\text{-}FPO(X)$ and $r\text{-}F\beta O(X)$). Then, $r\text{-}FRO(X)$, $r\text{-}FPO(X)$, and $r\text{-}F\beta O(X)$ are all fuzzy soft r -minimal structures.

Definition 3.4. Let $(X, \widetilde{M})$ be an $r\text{-}FMS$, $e \in E$, and $r \in I_0$. The fuzzy soft r -minimal interior and fuzzy soft r -minimal closure of f_A , denoted by $I_m(e, f_A, r)$ and $C_m(e, f_A, r)$, respectively, are defined as $I_m(e, f_A, r) = \bigcup \{g_B \in \widetilde{M}_{e,r} : g_B \sqsubseteq f_A, g_B \in \widetilde{M}_{e,r}\}$; $C_m(e, f_A, r) = \bigcap \{g_B \in \widetilde{M}_{e,r} : f_A \sqsubseteq g_B, g_B \in \widetilde{M}_{e,r}\}$.

Theorem 3.1. Let $(X, \widetilde{M})$ be an $r\text{-}FMS$ and $f_A, g_B \in \widetilde{M}_{e,r}$. Then, the following statements hold:

- (1) $I_m(e, f_A, r) \sqsubseteq f_A$ and if $f_A \in \widetilde{M}_{e,r}$, $I_m(e, f_A, r) = f_A$.
- (2) $f_A \sqsubseteq C_m(e, f_A, r)$ and if $f_A^c \in \widetilde{M}_{e,r}$, $C_m(e, f_A, r) = f_A$.
- (3) If $f_A \sqsubseteq g_B$, $I_m(e, f_A, r) \sqsubseteq I_m(e, g_B, r)$ and $C_m(e, f_A, r) \sqsubseteq C_m(e, g_B, r)$.
- (4) $I_m(e, f_A \sqcap g_B, r) = I_m(e, f_A, r) \sqcap I_m(e, g_B, r)$.
- (5) $C_m(e, f_A \sqcup g_B, r) = C_m(e, f_A, r) \sqcup C_m(e, g_B, r)$.
- (6) $I_m(e, I_m(e, f_A, r), r) = I_m(e, f_A, r)$ and $C_m(e, C_m(e, f_A, r), r) = C_m(e, f_A, r)$.
- (7) $(C_m(e, f_A, r))^c = I_m(e, f_A^c, r)$ and $(I_m(e, f_A, r))^c = C_m(e, f_A^c, r)$.

Proof. We prove (5) only because the others are obvious. For $f_A, g_B \in \widetilde{M}_{e,r}$, because $C_m(e, f_A, r) \sqsubseteq C_m(e, f_A \sqcup g_B, r)$ and $C_m(e, g_B, r) \sqsubseteq C_m(e, f_A \sqcup g_B, r)$, we obtain $C_m(e, f_A, r) \sqcup C_m(e, g_B, r) \sqsubseteq C_m(e, f_A \sqcup g_B, r)$. For $h_C^c, k_D^c \in \widetilde{M}_{e,r}$, $h_C^c \sqcap k_D^c$ does not always belong to $\widetilde{M}_{e,r}$. Thus, we obtain the following:

$$\begin{aligned} & C_m(e, f_A, r) \sqcup C_m(e, g_B, r) \\ &= (\bigcap \{h_C \in \widetilde{M}_{e,r} \mid h_C^c \in \widetilde{M}_{e,r}, f_A \sqsubseteq h_C\}) \\ &\quad \sqcup (\bigcap \{k_D \in \widetilde{M}_{e,r} \mid k_D^c \in \widetilde{M}_{e,r}, g_B \sqsubseteq k_D\}) \\ &= \bigcap \{h_C \sqcup k_D \in \widetilde{M}_{e,r} \mid f_A \sqsubseteq h_C, g_B \sqsubseteq k_D\} \\ &\sqsubseteq \bigcap \{w_E \in \widetilde{M}_{e,r} \mid w_E^c \in \widetilde{M}_{e,r}, f_A \sqcup g_B \sqsubseteq w_E\} \\ &= C_m(e, f_A \sqcup g_B, r). \end{aligned}$$

Hence, we obtain $C_m(e, f_A \sqcup g_B, r) = C_m(e, f_A, r) \sqcup C_m(e, g_B, r)$. $\square$

Definition 3.5. Let $(X, \widetilde{M})$ and $(Y, \widetilde{M}^*)$ be an $r\text{-}FMS$'s. Then, a fuzzy soft mapping φ_ψ from $(X, \widetilde{M})$ into $(Y, \widetilde{M}^*)$ is called fuzzy soft r -minimal continuous if $\varphi_\psi^{-1}(g_B) \in \widetilde{M}_{e,r}$ for every $g_B \in \widetilde{M}_{k,r}^*$, $e \in E$, and $(\psi(e) = k) \in F$.

Theorem 3.2. Let $\varphi_\psi : (X, \widetilde{M}) \rightarrow (Y, \widetilde{M}^*)$ be a fuzzy soft mapping, $f_A \in \widetilde{M}_{e,r}$, $g_B \in \widetilde{M}_{k,r}^*$, $e \in E$, and $r \in I_0$.

- (1) φ_ψ is fuzzy soft r -minimal continuous.
- (2) $(\varphi_\psi^{-1}(g_B))^c \in \widetilde{M}_{e,r}$ for every $g_B^c \in \widetilde{M}_{k,r}^*$.
- (3) $\varphi_\psi(C_m(e, f_A, r)) \sqsubseteq C_m^*(k, \varphi_\psi(f_A), r)$.
- (4) $C_m(e, \varphi_\psi^{-1}(g_B), r) \sqsubseteq \varphi_\psi^{-1}(C_m^*(k, g_B, r))$.
- (5) $\varphi_\psi^{-1}(I_m^*(k, g_B, r)) \sqsubseteq I_m(e, \varphi_\psi^{-1}(g_B), r)$.
- (6) If for every fuzzy soft point e_{x_t} over X and each $g_B \in \widetilde{M}_{k,r}^*$ containing $\varphi_\psi(e_{x_t})$, there exists $f_A \in \widetilde{M}_{e,r}$ containing e_{x_t} such that $\varphi_\psi(f_A) \sqsubseteq g_B$.

Then, (1) $\Leftrightarrow$ (2) $\Rightarrow$ (3) $\Leftrightarrow$ (4) $\Leftrightarrow$ (5) $\Rightarrow$ (6).

Proof. (1) $\Leftrightarrow$ (2) Obvious.

(2) $\Rightarrow$ (3) For all $f_A \in \widetilde{M}_{e,r}$,

$$\begin{aligned} & \varphi_\psi^{-1}(C_m^*(k, \varphi_\psi(f_A), r)) \\ &= \varphi_\psi^{-1}(\bigcap \{g_B \in \widetilde{M}_{k,r}^* \mid g_B^c \in \widetilde{M}_{k,r}^*, \varphi_\psi(f_A) \sqsubseteq g_B\}) \\ &= \bigcap \{\varphi_\psi^{-1}(g_B) \in \widetilde{M}_{e,r} \mid g_B^c \in \widetilde{M}_{k,r}^*, f_A \sqsubseteq \varphi_\psi^{-1}(g_B)\} \\ &\quad \sqsubseteq \bigcap \{\varphi_\psi^{-1}(g_B) \mid (\varphi_\psi^{-1}(g_B))^c \in \widetilde{M}_{e,r}, f_A \sqsubseteq \varphi_\psi^{-1}(g_B)\} \\ &= C_m(e, f_A, r). \end{aligned}$$

Hence $\varphi_\psi(C_m(e, f_A, r)) \sqsubseteq C_m^*(k, \varphi_\psi(f_A), r)$.

(3) $\Leftrightarrow$ (4) Let $g_B \in \widetilde{M}_{k,r}^*$. Then, from (3), it follows that $\varphi_\psi(C_m(e, \varphi_\psi^{-1}(g_B), r)) \sqsubseteq C_m^*(k, \varphi_\psi(\varphi_\psi^{-1}(g_B)), r) \sqsubseteq C_m^*(k, g_B, r)$. Hence, we obtain Equation (4). Similarly, we obtain (4) $\Rightarrow$ (3).

(4) $\Leftrightarrow$ (5) From (7) of Theorem 3.1.

(5) $\Rightarrow$ (6) Let $e_{x_t} \in \widetilde{P}_t(X)$ and $g_B \in \widetilde{M}_{k,r}^*$ with $\varphi_\psi(e_{x_t}) \in g_B$. From (5), it follows that $e_{x_t} \in I_m(e, \varphi_\psi^{-1}(g_B), r)$. Thus, there exists $f_A \in \widetilde{M}_{e,r}$ such that $e_{x_t} \in f_A \sqsubseteq \varphi_\psi^{-1}(g_B)$. Hence, we obtain (6). $\square$

Lemma 3.1. Let $\varphi_\psi : (X, \widetilde{M}) \rightarrow (Y, \widetilde{M}^*)$ be a fuzzy soft r -minimal continuous mapping. The following are then equivalent: For $e \in E$ and $r \in I_0$,

- (1) $\varphi_\psi(f_A) \sqsubseteq C_m^*(k, \varphi_\psi(f_A), r)$ for each $f_A \in \widetilde{M}_{e,r}$;
- (2) $\varphi_\psi^{-1}(g_B) = C_m(e, \varphi_\psi^{-1}(g_B), r)$ for each $g_B^c \in \widetilde{M}_{k,r}^*$;
- (3) $\varphi_\psi^{-1}(g_B) = I_m(e, \varphi_\psi^{-1}(g_B), r)$ for each $g_B \in \widetilde{M}_{k,r}^*$.

Proof. The proof is obvious. $\square$

Definition 3.6. Let X be a nonempty set and $\widetilde{M} : E \rightarrow [0, 1]^{\widetilde{(X, E)}}$. Then, $\widetilde{M}$ is said to have a property (P) if for $(f_A)_j \in \widetilde{(X, E)}$, $j \in J$, and $e \in E$

$$\widetilde{M}_e(\sqcup_{j \in J} (f_A)_j) \geq \sqcap_{j \in J} \widetilde{M}_e((f_A)_j).$$

Theorem 3.3. Let $(X, \widetilde{M})$ be an r -FMS with the property (P). Then, the following statements hold: (1) $I_m(e, f_A, r) = f_A$ iff $f_A \in \widetilde{M}_{e,r}$ for $f_A \in \widetilde{(X, E)}$, and (2) $C_m(e, f_A, r) = f_A$ if $f_A^c \in \widetilde{M}_{e,r}$ for $f_A \in \widetilde{(X, E)}$.

Proof. (1) From Theorem 3.1, it is sufficient to show that if $I_m(e, f_A, r) = f_A$ then $f_A \in \widetilde{M}_{e,r}$. Let $I_m(e, f_A, r) = f_A$, $e \in E$, and $r \in I_o$. Then, $\widetilde{M}_e(f_A) = \widetilde{M}_e(\sqcup \{g_B \in \widetilde{(X, E)} : g_B \sqsubseteq f_A, g_B \in \widetilde{M}_{e,r}\}) \geq \sqcap \widetilde{M}_e(g_B) \geq r$. Hence, $f_A \in \widetilde{M}_{e,r}$.

(2) From Theorem 3.1, it is sufficient to show that if $C_m(e, f_A, r) = f_A$, then $f_A^c \in \widetilde{M}_{e,r}$. Let $C_m(e, f_A, r) = f_A$, $e \in E$ and $r \in I_o$. Then, from Theorem 3.1, it follows that $f_A^c = (C_m(e, f_A, r))^c = I_m(e, f_A^c, r)$. Hence by (1), $f_A^c \in \widetilde{M}_{e,r}$. $\square$

Lemma 3.2. Let $\varphi_\psi : (X, \widetilde{M}) \rightarrow (Y, \widetilde{M}^*)$ be a fuzzy soft mapping and X have the property (P). Then, the following are equivalent: For $f_A \in \widetilde{(X, E)}$, $g_B \in \widetilde{(Y, F)}$, $e \in E$, and $r \in I_o$.

- (1) φ_ψ is fuzzy soft r -minimal continuous.
- (2) $(\varphi_\psi^{-1}(g_B))^c \in \widetilde{M}_{e,r}$ for every $(g_B)^c \in \widetilde{M}_{k,r}^*$.
- (3) $\varphi_\psi(C_m(e, f_A, r)) \sqsubseteq C_m^*(k, \varphi_\psi(f_A), r)$.
- (4) $C_m(e, \varphi_\psi^{-1}(g_B), r) \sqsubseteq \varphi_\psi^{-1}(C_m^*(k, g_B, r))$.
- (5) $\varphi_\psi^{-1}(I_m^*(k, g_B, r)) \sqsubseteq I_m(e, \varphi_\psi^{-1}(g_B), r)$.

Proof. The proof follows from Theorems 3.2 and 3.3. $\square$

Definition 3.7. Let $\varphi_\psi : (X, \widetilde{M}) \rightarrow (Y, \widetilde{M}^*)$ be a fuzzy soft mapping, $e \in E$ and $r \in I_o$. Then, φ_ψ is called

- (1) fuzzy soft r -minimal open if $\varphi_\psi(f_A) \in \widetilde{M}_{k,r}^*$ for every $f_A \in \widetilde{M}_{e,r}$, and
- (2) fuzzy soft r -minimal closed if $(\varphi_\psi(f_A))^c \in \widetilde{M}_{k,r}^*$ for every $f_A \in \widetilde{M}_{e,r}$.

Theorem 3.4. Let $\varphi_\psi : (X, \widetilde{M}) \rightarrow (Y, \widetilde{M}^*)$ be a fuzzy soft mapping, $f_A \in \widetilde{(X, E)}$, $g_B \in \widetilde{(Y, F)}$, $e \in E$, and $r \in I_o$.

- (1) φ_ψ is fuzzy soft r -minimal open.
- (2) $\varphi_\psi(I_m(e, f_A, r)) \sqsubseteq I_m^*(k, \varphi_\psi(f_A), r)$.
- (3) $I_m(e, \varphi_\psi^{-1}(g_B), r) \sqsubseteq \varphi_\psi^{-1}(I_m^*(k, g_B, r))$.

Then, (1) $\Rightarrow$ (2) $\Leftrightarrow$ (3).

Proof. (1) $\Rightarrow$ (2) For $f_A \in \widetilde{(X, E)}$,

$$\begin{aligned} & \varphi_\psi(I_m(e, f_A, r)) \\ &= \varphi_\psi(\sqcup \{h_C \in \widetilde{(X, E)} : h_C \sqsubseteq f_A, h_C \in \widetilde{M}_{e,r}\}) \\ & \sqsubseteq \sqcup \{\varphi_\psi(h_C) : \varphi_\psi(h_C) \sqsubseteq \varphi_\psi(f_A), \varphi_\psi(h_C) \in \widetilde{M}_{k,r}^*\} \\ & I_m^*(k, \varphi_\psi(f_A), r). \end{aligned}$$

Hence, $\varphi_\psi(I_m(e, f_A, r)) \sqsubseteq I_m^*(k, \varphi_\psi(f_A), r)$.

(2) $\Leftrightarrow$ (3) For $g_B \in \widetilde{(Y, F)}$, from (2), it follows that $\varphi_\psi(I_m(e, \varphi_\psi^{-1}(g_B), r)) \sqsubseteq I_m^*(k, \varphi_\psi(\varphi_\psi^{-1}(g_B)), r) \sqsubseteq I_m^*(k, g_B, r)$. Hence, we obtain (3). Similarly, we obtain (3) $\Rightarrow$ (2). $\square$

Theorem 3.5. Let $\varphi_\psi : (X, \widetilde{M}) \rightarrow (Y, \widetilde{M}^*)$ be a fuzzy soft r -minimal open. Then, $\varphi_\psi(f_A) = I_m^*(k, \varphi_\psi(f_A), r)$ for every $f_A \in \widetilde{M}_{e,r}$.

Proof. From Theorem 3.4(2), the proof is obvious. $\square$

Lemma 3.3. Let $\varphi_\psi : (X, \widetilde{M}) \rightarrow (Y, \widetilde{M}^*)$ be a fuzzy soft mapping and Y have the property (P). Then, the following are equivalent: For $f_A \in \widetilde{(X, E)}$, $g_B \in \widetilde{(Y, F)}$, $e \in E$, and $r \in I_o$.

- (1) φ_ψ is fuzzy soft r -minimal open.
- (2) $\varphi_\psi(I_m(e, f_A, r)) \sqsubseteq I_m^*(k, \varphi_\psi(f_A), r)$.
- (3) $I_m(e, \varphi_\psi^{-1}(g_B), r) \sqsubseteq \varphi_\psi^{-1}(I_m^*(k, g_B, r))$.

Proof. The proof follows from Theorems 3.3 and 3.5. $\square$

Theorem 3.6. Let $\varphi_\psi : (X, \widetilde{M}) \rightarrow (Y, \widetilde{M}^*)$ be a fuzzy soft mapping, $f_A \in \widetilde{(X, E)}$, $g_B \in \widetilde{(Y, F)}$, $e \in E$, and $r \in I_o$.

- (1) φ_ψ is fuzzy soft r -minimal closed.
- (2) $\varphi_\psi(C_m(e, f_A, r)) \sqsubseteq C_m^*(k, \varphi_\psi(f_A), r)$.
- (3) $C_m(e, \varphi_\psi^{-1}(g_B), r) \sqsubseteq \varphi_\psi^{-1}(C_m^*(k, g_B, r))$.

Then, (1) $\Rightarrow$ (2) $\Leftrightarrow$ (3).

Proof. (1) $\Rightarrow$ (2) For $f_A \in \widetilde{(X, E)}$,

$$\begin{aligned} & \varphi_\psi(C_m(e, f_A, r)) \\ &= \varphi_\psi(\sqcap \{h_C \in \widetilde{(X, E)} : f_A \sqsubseteq h_C, h_C^c \in \widetilde{M}_{e,r}\}) \\ & \sqsubseteq \sqcap \{\varphi_\psi(h_C) : \varphi_\psi(f_A) \sqsubseteq \varphi_\psi(h_C), (\varphi_\psi(h_C))^c \in \widetilde{M}_{k,r}^*\} \\ & C_m^*(k, \varphi_\psi(f_A), r). \end{aligned}$$

Hence, $\varphi_\psi(C_m(e, f_A, r)) \sqsubseteq C_m^*(k, \varphi_\psi(f_A), r)$.

(2) $\Leftrightarrow$ (3) For $g_B \in \widetilde{(Y, F)}$, from (2), it follows that $\varphi_\psi(C_m(e, \varphi_\psi^{-1}(g_B), r)) \sqsubseteq C_m^*(k, \varphi_\psi(\varphi_\psi^{-1}(g_B)), r) \sqsubseteq C_m^*(k, g_B, r)$. Hence, we obtain (3). Similarly, we obtain (3) $\Rightarrow$ (2). $\square$

4. Several Types of Fuzzy Soft r -Minimal Compactness

In this section, several types of fuzzy soft r -minimal compactness are defined, and the relationships between them are characterized.

Definition 4.1. Let $(X, \widetilde{M})$ be an r -FMS, $g_B \in (\widetilde{X}, \widetilde{E})$, $e \in E$, and $r \in I_0$. Then, g_B is called a fuzzy soft r -minimal compact iff for every family $\{(f_A)_i \in (\widetilde{X}, \widetilde{E}) \mid (f_A)_i \in \widetilde{M}_{e,r}\}_{i \in \Gamma}$ such that $g_B \sqsubseteq \bigsqcup_{i \in \Gamma} (f_A)_i$, there exists a finite subset Γ_0 of Γ such that $g_B \sqsubseteq \bigsqcup_{i \in \Gamma_0} (f_A)_i$.

Theorem 4.1. Let $\varphi_\psi : (X, \widetilde{M}) \longrightarrow (Y, \widetilde{M}^*)$ be fuzzy soft r -minimal continuous. If $f_A \in (\widetilde{X}, \widetilde{E})$ is fuzzy soft r -minimal compact, then $\varphi_\psi(f_A)$ is fuzzy soft r -minimal compact.

Proof. Let $\{(g_B)_i \in (\widetilde{Y}, \widetilde{F}) \mid (g_B)_i \in \widetilde{M}_{k,r}^*\}_{i \in \Gamma}$ with $\varphi_\psi(f_A) \sqsubseteq \bigsqcup_{i \in \Gamma} (g_B)_i$. Then, $\{\varphi_\psi^{-1}((g_B)_i) \in (\widetilde{X}, \widetilde{E}) \mid \varphi_\psi^{-1}((g_B)_i) \in \widetilde{M}_{e,r}\}_{i \in \Gamma}$ (by φ_ψ is fuzzy soft r -minimal continuous) such that $f_A \sqsubseteq \bigsqcup_{i \in \Gamma} \varphi_\psi^{-1}((g_B)_i)$. Because f_A is fuzzy soft r -minimal compact, there exists a finite subset Γ_0 of Γ such that $f_A \sqsubseteq \bigsqcup_{i \in \Gamma_0} \varphi_\psi^{-1}((g_B)_i)$. Hence, $\varphi_\psi(f_A) \sqsubseteq \bigsqcup_{i \in \Gamma_0} (g_B)_i$. $\square$

Definition 4.2. Let $(X, \widetilde{M})$ be an r -FMS, $g_B \in (\widetilde{X}, \widetilde{E})$, $e \in E$, and $r \in I_0$. Then, g_B is called fuzzy soft r -minimal almost compact iff for every family $\{(f_A)_i \in (\widetilde{X}, \widetilde{E}) \mid (f_A)_i \in \widetilde{M}_{e,r}\}_{i \in \Gamma}$ such that $g_B \sqsubseteq \bigsqcup_{i \in \Gamma} (f_A)_i$, there exists a finite subset Γ_0 of Γ such that $g_B \sqsubseteq \bigsqcup_{i \in \Gamma_0} C_m(e, (f_A)_i, r)$.

Lemma 4.1. Let $(X, \widetilde{M})$ be an r -FMS. If $f_A \in (\widetilde{X}, \widetilde{E})$ is fuzzy soft r -minimal compact, it is then also almost fuzzy soft r -minimal compact.

Proof. Obvious. $\square$

Example 4.1. Let $X = I$, $n \in N - \{1\}$, and $E = \{e_1, e_2\}$ be the parameter set of X . Define f_{E_n} and $g_{E_1} \in (\widetilde{X}, \widetilde{E})$ as follows $\forall e \in E$:

$$f_{E_n}(e)(x) = \begin{cases} 0.8, & \text{if } x = 0, \\ nx, & \text{if } 0 < x \leq \frac{1}{n}, \\ 1, & \text{if } \frac{1}{n} < x \leq 1, \end{cases}$$

$$g_{E_1}(e)(x) = \begin{cases} 1, & \text{if } x = 0, \\ \frac{1}{2}, & \text{otherwise.} \end{cases}$$

We define the fuzzy soft r -minimal structure $\widetilde{M}_E : E \longrightarrow$

$[0, 1]^{(\widetilde{X}, \widetilde{E})}$ as follows: $\forall e \in E$:

$$\widetilde{M}_e(w_E) = \begin{cases} \frac{4}{5}, & \text{if } w_E \in \{\Phi, \widetilde{E}\}, \\ \frac{n}{n+1}, & \text{if } w_E \leq f_{E_n}, \\ \frac{2}{3}, & \text{if } w_E \leq g_{E_1}, \\ 0, & \text{otherwise.} \end{cases}$$

Then, X is almost fuzzy soft $\frac{1}{2}$ -minimal compact but not fuzzy soft $\frac{1}{2}$ -minimal compact.

Theorem 4.2. Let $\varphi_\psi : (X, \widetilde{M}) \longrightarrow (Y, \widetilde{M}^*)$ be fuzzy soft r -minimal continuous. If $f_A \in (\widetilde{X}, \widetilde{E})$ is almost fuzzy soft r -minimal compact, $\varphi_\psi(f_A)$ is almost fuzzy soft r -minimal compact.

Proof. Let $\{(g_B)_i \in (\widetilde{Y}, \widetilde{F}) \mid (g_B)_i \in \widetilde{M}_{k,r}^*\}_{i \in \Gamma}$ with $\varphi_\psi(f_A) \sqsubseteq \bigsqcup_{i \in \Gamma} (g_B)_i$. Then, $\{\varphi_\psi^{-1}((g_B)_i) \in (\widetilde{X}, \widetilde{E}) \mid \varphi_\psi^{-1}((g_B)_i) \in \widetilde{M}_{e,r}\}_{i \in \Gamma}$ (by φ_ψ is fuzzy soft r -minimal continuous) such that $f_A \sqsubseteq \bigsqcup_{i \in \Gamma} \varphi_\psi^{-1}((g_B)_i)$. Because f_A is almost fuzzy soft r -minimal compact, a finite subset Γ_0 exists such that $f_A \sqsubseteq \bigsqcup_{i \in \Gamma_0} C_m(e, \varphi_\psi^{-1}((g_B)_i), r)$. From Theorem 3.2(4), it follows that

$$\begin{aligned} & \bigsqcup_{i \in \Gamma_0} C_m(e, \varphi_\psi^{-1}((g_B)_i), r) \\ & \sqsubseteq \bigsqcup_{i \in \Gamma_0} \varphi_\psi^{-1}(C_m^*(k, (g_B)_i, r)) \\ & = \varphi_\psi^{-1}(\bigsqcup_{i \in \Gamma_0} C_m^*(k, (g_B)_i, r)). \end{aligned}$$

Hence, $\varphi_\psi(f_A) \sqsubseteq \bigsqcup_{i \in \Gamma_0} C_m^*(k, (g_B)_i, r)$. $\square$

Definition 4.3. Let $(X, \widetilde{M})$ be an r -FMS, $g_B \in (\widetilde{X}, \widetilde{E})$, $e \in E$, and $r \in I_0$. Then, g_B is called nearly fuzzy soft r -minimal compact iff for every family $\{(f_A)_i \in (\widetilde{X}, \widetilde{E}) \mid (f_A)_i \in \widetilde{M}_{e,r}\}_{i \in \Gamma}$ such that $g_B \sqsubseteq \bigsqcup_{i \in \Gamma} (f_A)_i$, there exists a finite subset Γ_0 of Γ in which $g_B \sqsubseteq \bigsqcup_{i \in \Gamma_0} I_m(e, C_m(e, (f_A)_i, r))$.

Lemma 4.2. Let $(X, \widetilde{M})$ be an r -FMS. If $f_A \in (\widetilde{X}, \widetilde{E})$ is fuzzy soft r -minimal compact, then it is also nearly fuzzy soft r -minimal compact.

Proof. For any $f_A \in \widetilde{M}_{e,r}$ from Theorem 3.1, it follows that $f_A = I_m(e, f_A, r) \sqsubseteq I_m(e, C_m(e, f_A, r), r)$. Thus, we obtain the following result. $\square$

Example 4.2. Let $X = I$, $0 < n < 1$ and $E = \{e_1, e_2\}$ be the parameter set of X . Define f_{E_n} , f_E , and $g_E \in (\widetilde{X}, \widetilde{E})$ as

follows $\forall e \in E$:

$$f_{E_n}(e)(x) = \begin{cases} \frac{x}{n}, & \text{if } 0 \leq x \leq n, \\ \frac{1-x}{1-n}, & \text{if } n < x \leq 1, \end{cases}$$

$$f_E(e)(x) = \begin{cases} 1, & \text{if } x = 0, \\ \frac{1}{2}, & \text{if } 0 < x \leq 1, \end{cases}$$

$$g_E(e)(x) = \begin{cases} \frac{1}{2}, & \text{if } 0 \leq x < 1, \\ 1, & \text{if } x = 1. \end{cases}$$

We define the fuzzy soft r -minimal structure $\widetilde{M}_E : E \longrightarrow [0, 1]^{(X, E)}$ as follows: $\forall e \in E$:

$$\widetilde{M}_e(w_E) = \begin{cases} 1, & \text{if } w_E \in \{f_E, g_E, \Phi, \widetilde{E}\}, \\ \max(\{1 - n, n\}), & \text{if } w_E = f_{E_n}, \\ 0, & \text{otherwise.} \end{cases}$$

Then, X is nearly fuzzy soft $\frac{1}{2}$ -minimal compact but is not fully fuzzy soft $\frac{1}{2}$ -minimal compact.

Theorem 4.3. Let $\varphi_\psi : (X, \widetilde{M}) \longrightarrow (Y, \widetilde{M}^*)$ be fuzzy soft r -minimal continuous and fuzzy soft r -minimal open mapping. If $f_A \in (X, \widetilde{E})$ is nearly fuzzy soft r -minimal compact, then $\varphi_\psi(f_A)$ is nearly fuzzy soft r -minimal compact.

Proof. Let $\{(g_B)_i \in (\widetilde{Y}, \widetilde{F}) \mid (g_B)_i \in \widetilde{M}_{k,r}^*, i \in \Gamma\}$ with $\varphi_\psi(f_A) \sqsubseteq \bigsqcup_{i \in \Gamma} (g_B)_i$. Then, $\{\varphi_\psi^{-1}((g_B)_i) \in (X, \widetilde{E}) \mid \varphi_\psi^{-1}((g_B)_i) \in \widetilde{M}_{e,r}, i \in \Gamma\}$ (by φ_ψ is fuzzy soft r -minimal continuous) such that $f_A \sqsubseteq \bigsqcup_{i \in \Gamma} \varphi_\psi^{-1}((g_B)_i)$. Because f_A is nearly fuzzy soft r -minimal compact, there exists a finite subset Γ_o of Γ such that

$$f_A \sqsubseteq \bigsqcup_{i \in \Gamma_o} I_m(e, C_m(e, \varphi_\psi^{-1}((g_B)_i), r), r).$$

From Theorems 3.2, and 3.4, it follows that

$$\begin{aligned} \varphi_\psi(f_A) &\sqsubseteq \bigsqcup_{i \in \Gamma_o} \varphi_\psi(I_m(e, C_m(e, \varphi_\psi^{-1}((g_B)_i), r), r)) \\ &\sqsubseteq \bigsqcup_{i \in \Gamma_o} I_{m^*}(k, \varphi_\psi(C_m(e, \varphi_\psi^{-1}((g_B)_i), r)), r) \\ &\sqsubseteq \bigsqcup_{i \in \Gamma_o} I_{m^*}(k, \varphi_\psi(\varphi_\psi^{-1}(C_m^*(k, (g_B)_i, r))), r) \\ &\sqsubseteq \bigsqcup_{i \in \Gamma_o} I_{m^*}(k, C_m^*(k, (g_B)_i, r), r). \end{aligned}$$

Hence,

$$\varphi_\psi(f_A) \sqsubseteq \bigsqcup_{i \in \Gamma_o} I_{m^*}(k, C_m^*(k, (g_B)_i, r), r). \quad \square$$

5. Conclusion

Many researchers have studied fuzzy soft set theory, which is easily applied to many problems having uncertainties from social life. In this paper, we introduced and explored a new form of fuzzy r -minimal structure introduced by Yoo et al. [28] called a fuzzy soft r -minimal structure, which is an extension of the fuzzy soft topology introduced by Aygunoglu et al. [26]. First, the concepts of fuzzy soft r -minimal continuity and fuzzy soft r -minimal compactness were introduced and studied in fuzzy soft r -minimal spaces. Finally, several types of fuzzy soft r -minimal compactness were defined, and the following implications were found to hold: fuzzy soft r -minimal compact $\Rightarrow$ fuzzy soft r -minimal nearly compact $\Rightarrow$ fuzzy soft r -minimal almost compact. We hope that the findings of this study will help researchers enhance and promote further studies on fuzzy soft r -minimal spaces and carry out a general framework for their applications in other scientific areas.

Conflict of Interest

No potential conflicts of interest relevant to this article are reported.

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