Enhancing Zero-Based Budgeting Under Fuzzy Environment

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Abstract

Zero-based budgeting (ZBB) is a well-known method for the selection and management of budgets and is widely used by companies and government agencies. In this paper, a new method for ZBB modelling using fuzzy numbers is described. Imprecise budget data are described using pentagonal fuzzy numbers. This method is best illustrated through a numerical example.

Keywords: Zero-based budgeting, Pentagonal fuzzy number, Maximal level, Risk, Fuzzy threshold

1. Introduction

Zero-based budgeting (ZBB) is a method of budgeting in which all expenses for the new period are calculated based on actual expenses that are incurred and not on a differential basis, which involves simply changing the expenses incurred by taking into account changes in operational activity. Under this method, every activity needs to be justified, explaining the revenue that all costs will generate for the company, which involves a re-evaluation of every line item of the cash flow statement and justifying all expenditures incurred by a department.

In [1], the ZB design was introduced for public organizations in a step-by-step manner. Pyhrr [2] identified four basic steps that constitute the ZBB process.

Windsor [3] suggested two theories for ZBB as well as a separate theory of a zero-based review (ZBR) in the public sector. One theory of ZBB deals with indirect costs in business or government. Another theory of ZBB deals with discretionary spending in government because of the environmental differences between the private and public sectors. In addition, a theory for ZBR is needed for non-discretionary spending in government. Joshi [4] presented a survey of the current and potential users of ZBB, and analyzing the perceptions of the various factors, found that management-oriented aspects have to be considered by Indian enterprises in the design and effective implementation of ZBB. Ahmed [5] investigated the perceptions and attitudes of employees in selected public sector organizations in Brunei Darussalam toward the adoption of ZBB. In addition, Shayne [6] confirmed that governments across the global are facing budget cuts and increased public scrutiny, and that government agencies have used alternative budgeting methods such as ZBB instead of line items and incremental budgeting. The author suggests that the ZBB process should be computerized to minimize the problems

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arising in its implementation. Ibrahim et al. [7] also conducted a study on predicting the possibility of adopting a ZBB system in the state of Borno when considering viability as a predictor, which has been perceived to have contributed to the adoption of the ZBB. Moreover, Alamry et al. [8] used a zero-budget system in the sub-units in the government of Iraq.

Fuzzy set theory plays an important role in uncertainty modeling. Zadeh [9] first proposed the philosophy of fuzzy sets. Decision making in a fuzzy environment, developed by Bellman and Zadeh [10], has improved management decision problems. Zimmermann [11] introduced fuzzy and linear programming with multiple objective functions. Later, several researchers worked on fuzzy set theory. Dubois and Prade [12] studied the theory and applications of fuzzy sets and systems. Kaufmann and Gupta [13] also studied several fuzzy mathematical models along with their applications in the engineering and management sciences. Selvam et al. [14] introduced five different points of pentagonal fuzzy numbers, new operations, the ranking of pentagonal fuzzy numbers, and the application of a centroid incenter. Kamble [15] discussed the basic concepts of pentagonal fuzzy numbers using internal arithmetic operations using α -cut operations. Ghosh et al. [16] used the geographic information system, multiple-criteria decision-making tools, fuzzy analytic hierarchy process (FAHP), fuzzy technique for order of preference by similarity to ideal solution (FTOPSIS), and a fuzzy complex proportional assessment to obtain the optimal site selection for an EV charging station. In addition, Ghorui et al. [17] evaluated the risk factors involved in the spread of COVID-19 and ranked such factors using the FAHP and hesitant fuzzy sets when applying FTOPSIS. Tudu et al. [18] also proposed a new representation of type-2 fuzzy numbers, namely, generalized type-2 fuzzy numbers, and developed theorems for solving generalized type-2 fuzzy boundary value problems. Rahaman et al. [19] applied a Gaussian method to solve the linear difference equation in a fuzzy environment. In addition, Ghorui et al. [20] applied FAHP and FTOPSIS for shopping mall site selection, and Ghosh et al. [21] applied FAHP and FTOPSIS for selecting the best e-rickshaw available. Moreover, Abtahi et al. [22] proposed a skew-normal uncertainty distribution to capture the skewness in the portfolio selection problem, and Fazlollohtabar and Ghlizadeh [23] studied a single-server finite-capacity Markovian queuing system with encouraged arrivals. Kaufman [24] extended the ZBB approach to a fuzzy environment, introduced a numerical example to explain how to use this extended tool, and described the selection procedure. Kalantari et al. [25] presented a mathematical model designed

using Chebyshev's goal programming technique with a fuzzy approach for performance-based budgeting and combined it with a rolling budget for increased flexibility. Finally, Khalifa et al. [26] proposed a new technique for fuzzy portfolio selection.

In this study, a new method for modeling ZBB using fuzzy numbers is proposed. Imprecise data are described using pentagonal fuzzy numbers. The main contributions when applying ZBB or fuzzy zero-base budgeting (FZBB) are as follows:

1. one or more decision centers may be eliminated;
2. only the most efficient departments will receive a healthy budget; and
3. an optimum use of available resources is obtained.

The remainder of this article is organized as follows: Section 2 addresses some of the preliminaries and notations needed. Section 3 describes a numerical example. Section 4 discusses the results of this study. Finally, some concluding remarks are presented in Section 5.

2. Preliminaries

To discuss the present problem, the basic rules and findings related to a fuzzy set, fuzzy numbers, pentagonal fuzzy numbers, and arithmetic operations of pentagonal fuzzy numbers and their ranking should first be reviewed.

Definition 1 ([30]). Let $M = X_1 \times X_2 \times \cdots \times X_n$ be a Cartesian product, π_i be the fuzzy set in X_i , and $f : X \rightarrow Y$ be a mapping. Fuzzy set $\tilde{A}$ in Y can then be defined using the extension principle as follows:

$$\tilde{A}(y) = \begin{cases} \sup_{(x_1, \dots, x_n) \in f^{-1}(y)} \min\{\pi_1(x_1), \dots, \pi_n(x_n)\}, & f^{-1}(y) \neq \emptyset, \\ 0, & f^{-1}(y) = \emptyset. \end{cases}$$

Definition 2 ([30]). Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy sets, and define their algebraic operations as follows:

1. **Addition:** $\tilde{A}(+) \tilde{B}$

$$\pi_{\tilde{A}(+) \tilde{B}}(z) = \sup_{z=x+y} \min\{\pi_{\tilde{A}}, \pi_{\tilde{B}}\}, \quad x \in \tilde{A}, \quad y \in \tilde{B}.$$

2. **Subtraction:** $\tilde{A}(-) \tilde{B}$

$$\pi_{\tilde{A}(-) \tilde{B}}(z) = \sup_{z=x-y} \min\{\pi_{\tilde{A}}, \pi_{\tilde{B}}\}, \quad x \in \tilde{A}, \quad y \in \tilde{B}.$$

3. Multiplication: $\tilde{A}(\cdot)\tilde{B}$

$$\pi_{\tilde{A}(\cdot)\tilde{B}}(z) = \sup_{z=x \cdot y} \min\{\pi_{\tilde{A}}, \pi_{\tilde{B}}\}, x \in \tilde{A}, y \in \tilde{B}.$$

4. Division: $\tilde{A}(/)\tilde{B}$

$$\pi_{\tilde{A}(/)\tilde{B}}(z) = \sup_{z=x/y} \min\{\pi_{\tilde{A}}, \pi_{\tilde{B}}\}, x \in \tilde{A}, y \in \tilde{B}.$$

Definition 3 (Fuzzy number [9]). A fuzzy number $\tilde{E}$ is a fuzzy set with a membership function defined as

$\pi_{\tilde{E}}(x) : \mathfrak{R} \rightarrow [0, 1]$, where the following are satisfied:

1. $\tilde{E}$ is fuzzy convex, i.e., $\pi_{\tilde{E}}(\delta x + (1-\delta)y) \geq \min\{\pi_{\tilde{E}}(x), \pi_{\tilde{E}}(y)\}; \forall x, y \in \mathfrak{R}; 0 \leq \delta \leq 1$;
2. $\tilde{E}$ is normal, i.e., $\exists x_0 \in \mathfrak{R}$ for which $\pi_{\tilde{E}}(x_0) = 1$;
3. $\text{Supp}(\tilde{E}) = \{x \in \mathfrak{R} : \pi_{\tilde{E}}(x) > 0\}$ is the support of $\tilde{E}$; and
4. $\pi_{\tilde{E}}(x)$ is upper semi-continuous (that is, for each $\alpha \in (0, 1)$, the α -cut set $\tilde{E}_\alpha = \{x \in \mathfrak{R} : \pi_{\tilde{E}} \geq \alpha\}$ is closed).

Definition 4 (Pentagonal fuzzy number [29]). A pentagonal fuzzy number $\tilde{E}_P = (a_1, a_2, a_3, a_4, a_5)$ should satisfy the following conditions:

1. $\pi_{\tilde{E}_P}(x)$ is a continuous function in $[0, 1]$;
2. $\pi_{\tilde{E}_P}(x)$ is strictly increasing and is a continuous function on $[a_1, a_2]$ and $[a_2, a_3]$;
3. $\pi_{\tilde{E}_P}(x)$ is strictly decreasing and is a continuous function on $[a_3, a_4]$ and $[a_4, a_5]$.

Definition 5 (A linear pentagonal fuzzy number with symmetry [27–29]). A linear pentagonal fuzzy number is written as $\tilde{E}_P = (a_1, a_2, a_3, a_4, a_5)$, $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5$, on $\mathfrak{R}$ whose membership function is given by the following (Figure 1):

$$\pi_{\tilde{E}_P}(x) = \begin{cases} 0, & x < a_1, \\ w_1 \left(\frac{x-a_1}{a_2-a_1} \right), & \text{for } a_1 \leq x \leq a_2, \\ 1 - (1-w_1) \left(\frac{x-a_2}{a_3-a_2} \right), & \text{for } a_2 \leq x \leq a_3, \\ 1, & \text{for } x = a_3, \\ 1 - (1-w_2) \left(\frac{a_4-x}{a_4-a_3} \right), & \text{for } a_3 \leq x \leq a_4, \\ w_2 \left(\frac{a_5-x}{a_5-a_4} \right), & \text{for } a_4 \leq x \leq a_5, \\ 0, & \text{for } x > a_5. \end{cases}$$

Figure 1. Graphical representation of pentagonal Fuzzy number [14].

Definition 6. Let $\tilde{E}_P = (a_1, a_2, a_3, a_4, a_5)$, and $\tilde{F}_P = (b_1, b_2, b_3, b_4, b_5)$ be two pentagonal fuzzy numbers, $\gamma_b \neq 0$. The arithmetic operations on $\tilde{E}_P$ and $\tilde{F}_P$ are as follows:

- Addition:** $\tilde{E}_P \oplus \tilde{F}_P = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5)$, with $w_i(\tilde{E}_P \oplus \tilde{F}_P) \geq \max(w_i(\tilde{E}_P), w_i(\tilde{F}_P))$, $i = 1, 2$,
- Subtraction:** $\tilde{E}_P \ominus \tilde{F}_P = (a_1-b_5, a_2-b_4, a_3-b_3, a_4-b_2, a_5-b_1)$,
- Multiplication:** $\tilde{E}_P \otimes \tilde{F}_P = \frac{1}{5} \gamma_b (a_1, a_2, a_3, a_4, a_5)$, $\gamma_b = (b_1+b_2+b_3+b_4+b_5)$,
- Division:** $\frac{\tilde{E}_P}{\tilde{F}_P} = \frac{5}{\gamma_b} (a_1, a_2, a_3, a_4, a_5)$, $0 \neq \gamma_b = (b_1+b_2+b_3+b_4+b_5)$,
- Scalar multiplication:** $k\tilde{E}_P = \begin{cases} (ka_1, ka_2, ka_3, ka_4, ka_5), & k > 0, \\ (ka_5, ka_4, ka_3, ka_2, ka_1), & k < 0, \end{cases}$
- Inverse:** $\tilde{E}_P^{-1} = \left(\frac{1}{a_5}, \frac{1}{a_4}, \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1} \right)$.

Definition 7. The associated ordinary number corresponding to the pentagonal fuzzy number $\tilde{E}_P = (a_1, a_2, a_3, a_4, a_5)$ is defined by

$$R(\tilde{E}_P) = \hat{E} = \frac{a_1 + a_2 + 2a_3 + a_4 + a_5}{6}.$$

2.1 Notation

In ZBB, the following notations may be used.

$\tilde{A}_P(a_1, a_2, a_3, a_4, a_5)$: Pentagonal fuzzy number.

$a_1, a_5; a_2, a_4$: Bounds of presumption to least level ($\alpha = 0$ and $\alpha = 0.5$).

a_3 : The mode of presumption to maximal level ($\alpha = 1$).

$\tilde{H}$: Fuzzy threshold on available resources.

Figure 2. Four decision centers in a company for zero-based budgeting process.

3. Numerical Example

Consider a company with four decision centers, A, B, C, and D, as illustrated in Figure 2. Assume that the three possible budgets, $A_0 < A_1 < A_2$, have been defined for A; two possible budgets, $B_0 < B_1$, have been defined for B; four possible budgets, $C_0 < C_1 < C_2 < C_3$, have been defined for C; and three possible budgets, $D_0 < D_1 < D_2$, have been defined for D. Budgets with index 0 represent the minimal budget (i.e., under this allocation, the center will be deleted), budgets with index 1 represent normal budgets, whereas indices 2, 3, ..., n are improved budgets. In the most frequent cases, budgets are defined and established for one year at a time.

The ordered budgets can be represented by a flow graph, as illustrated in Figure 2. The decision-making group will choose the budget beginning with index 0. If a budget of index j is used for a center, it includes the budget on index i , $i < j$, as well.

Let us suppose that the budget selection will be as shown in Figure 3. Here, we assign a subjective assignment to each budget using pentagonal fuzzy numbers. Thus, 12 budgets may be assigned using pentagonal fuzzy numbers, as follows:

$$\begin{aligned} A_0 &= (900, 1000, 1100, 1250, 1400), \\ A_1 &= (1100, 1200, 1250, 1350, 1450), \\ A_2 &= (1400, 1500, 1600, 1750, 1900), \\ B_0 &= (700, 800, 1000, 1100, 1300), \\ B_1 &= (1100, 1300, 1400, 1500, 1700), \\ C_0 &= (300, 400, 500, 600, 700), \end{aligned}$$

Figure 3. Various steps in budget selection process.

$$\begin{aligned} C_1 &= (400, 500, 650, 750, 800), \\ C_2 &= (500, 650, 800, 900, 950), \\ C_3 &= (600, 800, 1000, 1100, 1200), \\ D_0 &= (900, 1000, 1200, 1300, 1400), \\ D_1 &= (1000, 1250, 1400, 1550, 1700), \\ D_2 &= (1300, 1500, 1700, 1800, 1900). \end{aligned} \quad (1)$$

In this cumulative budgeting process, the lower category from any decision center is dropped.

Steps:

- $C_0 = (300, 400, 500, 600, 700)$,
- $C_0 \oplus D_0 = (300, 400, 500, 600, 700) \oplus (900, 1000, 1200, 1300, 1400) = (1200, 1400, 1700, 1900, 2100)$,

- c. $C_1 \oplus D_0 = (400, 500, 650, 750, 800)$
 $\oplus (900, 1000, 1200, 1300, 1400)$
 $= (1300, 1500, 1850, 2050, 2200),$
- d. $A_0 \oplus B_1 \oplus D_0 = (2900, 3300, 3700, 4050, 4500),$
- e. $A_1 \oplus C_1 \oplus D_0 = (2400, 2700, 3100, 3400, 3650),$
- f. $A_1 \oplus B_0 \oplus C_1 \oplus D_0 = (3100, 3500, 4100, 4500, 4950),$
- g. $A_2 \oplus B_0 \oplus C_1 \oplus D_0 = (3400, 3800, 4450, 4900, 5400),$
- h. $A_2 \oplus B_0 \oplus C_2 \oplus D_0 = (3500, 3950, 4600, 5050, 5550),$
- i. $A_2 \oplus B_1 \oplus C_2 \oplus D_0 = (3900, 4450, 5000, 5450, 6450),$
- j. $A_2 \oplus B_1 \oplus C_3 \oplus D_0 = (4000, 4600, 5200, 5650, 6200),$
- k. $A_2 \oplus B_1 \oplus C_3 \oplus D_1 = (4100, 4850, 5400, 5900, 6500),$
- l. $A_2 \oplus B_1 \oplus C_3 \oplus D_2 = (4400, 5100, 5700, 6150, 6750).$
- (2)

From Eq. (2), it is clear that the value resulting from the proposed approach is greater than the values found in [13].

Because the total budget available to the company is limited, a threshold must be introduced instead of taking a deterministic number for this threshold, and thus we take a fuzzy threshold $\tilde{H}$, which is defined as

$$\forall x \in \mathfrak{R}^+ :$$

$$\mu_{\tilde{H}}(x) = \begin{cases} 1, & x \leq 5000, \\ \frac{6000 - x}{1000}, & 5000 \leq x \leq 6000, \\ 0, & x \geq 6000. \end{cases} \quad (3)$$

As with any budget, it is difficult to develop a total budget with precision. Therefore, a more realistic approach using fuzzy dat.

To establish a cumulative budget that can be agreed upon, we compute the possibility of each cumulative budget, taking $\tilde{H}$ as the “law of possibility.”

Definition 8. The possibility of X is defined as follows:

$$\forall x \in \mathfrak{R}^+ :$$

$$\text{Poss}(X) = \bigvee_x (\mu_X(x) \wedge \mu_{\tilde{H}}(x)). \quad (4)$$

Based on Definition 8, the cumulative budget is as follows:

- $\text{Poss}(A_2 \oplus B_0 \oplus C_2 \oplus D_0) = 1$, cumulative budget h;
 $\text{Poss}(A_2 \oplus B_1 \oplus C_2 \oplus D_0) = 1$, cumulative budget i;
 $\text{Poss}(A_2 \oplus B_1 \oplus C_3 \oplus D_0) = .87$, cumulative budget j;
 $\text{Poss}(A_2 \oplus B_1 \oplus C_3 \oplus D_1) = .74$, cumulative budget k;

$\text{Poss}(A_2 \oplus B_1 \oplus C_3 \oplus D_2) = .53$, cumulative budget l.

4. Results and Discussion

We found that budgets h and I can be agreed upon without any restriction with a possibility equal to 1; however, budget j is available but with a small risk (i.e., $\text{Poss}(j) = .87$). The possibilities of k and l are .74 and .53, respectively, and therefore there is a large risk associated with these budgets. Instead of the possibility, another good criterion, namely, the agreement risk defined in [31], can be used.

5. Conclusions and Future Research

In this paper, we showed the use of fuzzy numbers and possibility theory, as well as a novel method for modeling ZBB in a fuzzy environment, in which one or several decision centers may be eliminated. Thus, only the departments that are most efficient will receive a healthy budget, giving an optimum use of available resources. It is clear from the budgeting example that we can use fuzzy numbers and the possibility theory. In addition, this procedure can be applied to many similar problems.

Conflicts and Interest

The author declares no conflict of interest.

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