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Double-Framed Soft Deductive System of Subtraction Algebras

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Abstract

The notion of double-framed soft deductive system in subtraction algebras is introduced, and several properties are investigated. Conditions for a double-framed soft set to be a double-framed soft deductive system are provided. Characterizations of a double-framed soft deductive system are established.

Keywords: Subtraction algebra, Soft set, Double-framed soft set, Double-framed soft deductive system

1. Introduction

Zadeh [1] introduced the concept of fuzzy set as a new mathematical tool for dealing with uncertainties and so many researchers were conducted on the generalizations of the notion of fuzzy sets. Various theories have their own difficulties which are pointed out in [2]. To overcome these difficulties, Molodtsov [2] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Maji et al. [3, 4] also studied several operations on the theory of soft sets and described the application of soft set theory to a decision-making problem. Aktas and Cagman [5] studied the basic concepts of soft sets in ideal theory of *BCK/BCI*-algebras. We refer the reader to the papers [7–15] for further information regarding algebraic structures/properties of soft set theory.

Schein [16] have considered systems of the form $(\Phi; \circ, \backslash)$, where Φ is a set of functions closed under the composition " \circ " of functions (and hence $(\Phi; \circ)$ is a function semigroup) and the set theoretic subtraction " \backslash " (and hence $(\Phi; \backslash)$ is a subtraction algebra in the sense of [17]). Jun and Kim [18] discussed ideal theory of subtraction algebras.

In this paper we introduce a notion of double-framed soft deductive system in a subtraction algebra, and study some properties of them. We give conditions for a double-framed soft set to be a double-framed soft deductive system and establish characterizations of a double-framed soft deductive system

2. Preliminaries

A subtraction algebra is a system (X; -) with a single binary operation "-" that satisfies the following identities: for any $x, y, z \in X$,

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$$x - (y - x) = x; \tag{S1}$$

$$x - (x - y) = y - (y - x);$$
 (S2)

$$(x - y) - z = (x - z) - y.$$
 (S3)

The last identity permits us to omit parentheses in expressions of the form (x - y) - z. For any $a, b \in X$, we define an order relation \leq on X: $a \leq b \Leftrightarrow a - b = 0$, where 0 = a - ais an element that does not depend on the choice of $a \in X$. The ordered set $(X; \leq)$ is a semi-Boolean algebra in the sense of [17], that is, it is a meet semilattice with zero 0 in which every interval [0, a] is a Boolean algebra with respect to the induced order. Here $a \wedge b = a - (a - b)$; the complement of an element $b \in [0, a]$ is a - b; and if $b, c \in [0, a]$, then

$$b \lor c = (b' \land c')' = a - ((a - b) \land (a - c))$$

= a - ((a - b) - ((a - b) - (a - c)))

In a subtraction algebra, the following are true (see [18]):

$$(x-y) - y = x - y.$$
 (a1)

$$x - 0 = x$$
 and $0 - x = 0$. (a2)

$$(x - y) - x = 0. (a3)$$

$$x - (x - y) \le y. \tag{a4}$$

$$(x - y) - (y - x) = x - y.$$
 (a5)

$$x - (x - (x - y)) = x - y.$$
 (a6)

$$(x-y) - (z-y) \le x - z.$$
 (a7)

$$x \le y$$
 if and ony if $x = y - w$ for some $w \in X$. (a8)

$$x \le y$$
 implies $x - z \le y - z$ and $z - y \le z - x$

for all
$$z \in X$$
. (a9)

$$x, y \le z \text{ implies } x - y = x \land (z - y).$$
 (a10)

$$(x \wedge y) - (x \wedge z) \le x \wedge (y - z). \tag{a11}$$

Proposition 2.1 ([18]). Let X be a subtraction algebra and let $x, y \in X$. If $w \in X$ is an upper bound for x and y, then the element

$$x \lor y := w - ((w - y) - x)$$

is a least upper bound for x and y.

Definition 2.2 ([18]). A nonempty subset S of X is called a subalgebra of X if it satisfies :

$$(\forall x, y \in S) \ (x - y \in S).$$

Definition 2.3. A nonempty subset D of X is called a *deductive* system of X (it is called an *ideal* of X in [18]) if it satisfies:

- (1) $0 \in D$,
- (2) $(\forall x \in X)(\forall y \in D)(x y \in D \Rightarrow x \in D).$

Molodtsov [2] defined the soft set in the following way: Let U be an initial universe set and E be a set of parameters. Let $\mathscr{P}(U)$ denotes the power set of U and $A, B, C, \dots \subseteq E$.

Definition 2.4 ([2]). A pair $(\tilde{\alpha}, A)$ is called a *soft set* over U, where $\tilde{\alpha}$ is a mapping given by

$$\tilde{\alpha}: A \to \mathscr{P}(U).$$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $\tilde{\alpha}(\varepsilon)$ may be considered as the set of ε -approximate elements of the soft set $(\tilde{\alpha}, A)$. Clearly, a soft set is not a set. For illustration, Molodtsov [2] considered several examples in [2].

Definition 2.5 ([10]). A double-framed pair $\langle (\tilde{\alpha}, \tilde{\beta}); A \rangle$ is called a *double-framed soft set* of A over U, where $\tilde{\alpha}$ and $\tilde{\beta}$ are mappings from A to $\mathscr{P}(U)$.

3. Double-Framed Soft Deductive Systems

In what follows, we take E = X, as a set of parameters, which is a subtraction algebra and A, B, C, \cdots be subalgebras of X unless otherwise specified.

Definition 3.1. A double-framed soft $\langle (\tilde{\alpha}, \tilde{\beta}); X \rangle$ of X is called a double frame soft deductive system set of X over U if the following conditions are true:

$$(\forall x \in X) \ (\tilde{\alpha}(0) \supseteq \tilde{\alpha}(x), \ \tilde{\beta}(0) \subseteq \tilde{\beta}(x)),$$
 (c1)
$$(\forall x, y \in X) \ (\tilde{\alpha}(x) \supseteq \tilde{\alpha}(x-y) \cap \tilde{\alpha}(y),$$

$$\tilde{\beta}(x) \subseteq \tilde{\beta}(x-y) \cup \tilde{\beta}(y)),$$
 (c2)

Example 3.2. Consider a subtraction algebra

$$X = \{0, e_1, e_2, e_3, e_4\}$$

with the following Cayley table [19]:

_	0	e_1	e_2	e_3	e_4
0	0	0	0	0	0
e_1	e_1	0	e_1	0	e_1
e_2	e_2	e_2	0	0	e_2
e_3	e_3	e_2	e_1	0	e_3
e_4	e_4	e_4	e_4	e_4	0

For A = X and $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$. We define a double-framed soft set $\langle (\tilde{\alpha}, \tilde{\beta}); A \rangle$ over U as follows:

$$\tilde{\alpha}: A \to \mathscr{P}(U), \ x \mapsto \begin{cases} \{h_1, h_3, h_4, h_5\} & \text{if } x = 0, \\ \{h_1, h_3, h_4, h_5\} & \text{if } x = e_1, \\ \{h_3, h_4, h_5\} & \text{if } x = e_2, \\ \{h_3, h_4, h_5\} & \text{if } x = e_3, \\ \{h_1, h_3, h_4, h_5\} & \text{if } x = e_4, \end{cases}$$

and

$$\tilde{\beta}: A \to \mathscr{P}(U), \ x \mapsto \begin{cases} \{h_1, h_3\} & \text{if } x = 0, \\ \{h_1, h_3\} & \text{if } x = e_1, \\ \{h_1, h_3, h_4\} & \text{if } x = e_2 \\ \{h_1, h_3, h_4\} & \text{if } x = e_3, \\ \{h_1, h_3\} & \text{if } x = e_4. \end{cases}$$

It is routine to verify that $\langle (\tilde{\alpha}, \tilde{\beta}); A \rangle$ is a double-framed soft deductive system of X over U.

Proposition 3.3. Every double-framed soft deductive system $\langle (\tilde{\alpha}, \tilde{\beta}); X \rangle$ of X over U satisfies:

$$(\forall x, y \in X) \, (x \le y \Rightarrow \tilde{\alpha}(x) \supseteq \tilde{\alpha}(y), \tilde{\beta}(x) \subseteq \tilde{\beta}(y)). \quad (1)$$

Proof. Let $x, y \in X$ be such that $x \leq y$. Then x - y = 0, and so

$$\begin{split} \tilde{\alpha}(x) &\supseteq \tilde{\alpha}(x-y) \cap \tilde{\alpha}(y) = \tilde{\alpha}(0) \cap \tilde{\alpha}(y) = \tilde{\alpha}(y), \\ \tilde{\beta}(x) &\subseteq \tilde{\beta}(x-y) \cup \tilde{\beta}(y) = \tilde{\beta}(0) \cup \tilde{\beta}(y) = \tilde{\beta}(y). \end{split}$$

Proposition 3.4. Every double-framed soft deductive system $\langle (\tilde{\alpha}, \tilde{\beta}); X \rangle$ of X over U satisfies:

$$(\forall x, y, z \in X) \left(\begin{array}{c} \tilde{\alpha}(x-z) \supseteq \tilde{\alpha}((x-y)-z) \cap \tilde{\alpha}(y), \\ \tilde{\beta}(x-z) \subseteq \tilde{\beta}((x-y)-z) \cup \tilde{\beta}(y). \end{array} \right)$$
(2)

Proof. Combining (S3) and Definition 3.1 (c2), we get

$$\begin{split} \tilde{\alpha}(x-z) &\supseteq \tilde{\alpha}((x-z)-y) \cap \tilde{\alpha}(y) \\ &= \tilde{\alpha}((x-y)-z) \cap \tilde{\alpha}(y), \end{split}$$

and

$$\begin{split} \tilde{\beta}(x-z) &\subseteq \tilde{\beta}((x-z)-y) \cup \tilde{\beta}(y) \\ &= \tilde{\beta}((x-y)-z) \cup \tilde{\beta}(y) \end{split}$$

for all $x, y, z \in X$.

We give conditions for a double-framed soft set to be a double-framed soft deductive system .

Theorem 3.5. If double-framed soft set $\langle (\tilde{\alpha}, \tilde{\beta}); X \rangle$ of X over U satisfying (c1) and (2), then $\langle (\tilde{\alpha}, \tilde{\beta}); X \rangle$ is a double-framed soft deductive system of X over U.

Proof. Taking z = 0 in (2) and using (a2), we get

$$\tilde{\alpha}(x) = \tilde{\alpha}(x-0) \supseteq \tilde{\alpha}((x-y)-0) \cap \tilde{\alpha}(y)$$
$$= \tilde{\alpha}(x-y) \cap \tilde{\alpha}(y)$$

and

$$\begin{split} \tilde{\beta}(x) = &\tilde{\beta}(x-0) \subseteq \tilde{\beta}((x-y)-0) \cup \tilde{\beta}(y) \\ = &\tilde{\beta}(x-y) \cup \tilde{\beta}(y) \end{split}$$

for all $x, y \in X$. Hence $\left\langle (\tilde{\alpha}, \tilde{\beta}); X \right\rangle$ is a double-framed soft deductive system of X over U.

Corollary 3.6. Let $\langle (\tilde{\alpha}, \tilde{\beta}); X \rangle$ be a double-framed soft set of X over U. Then $\langle (\tilde{\alpha}, \tilde{\beta}); X \rangle$ is a double-framed soft deductive system of X over U if and only if it satisfies conditions (c1) and (2).

The following is a characterization of a double-framed soft deductive system of X over U.

Theorem 3.7. Let $\langle (\tilde{\alpha}, \tilde{\beta}); X \rangle$ be a double-framed soft set of X over U. Then $\langle (\tilde{\alpha}, \tilde{\beta}); X \rangle$ is a double-framed soft deductive system of X over U if and only if it satisfies the following conditions:

$$\begin{aligned} (\forall x, y \in X)(\tilde{\alpha}(x-y) \supseteq \tilde{\alpha}(x), \tilde{\beta}(x-y) \subseteq \tilde{\beta}(x)), \qquad (3) \\ (\forall x, a, b \in X) \left(\begin{array}{c} \tilde{\alpha}(x-((x-a)-b)) \supseteq \tilde{\alpha}(a) \cap \tilde{\alpha}(b), \\ \tilde{\beta}(x-((x-a)-b)) \subseteq \tilde{\beta}(a) \cup \tilde{\beta}(b) \end{array} \right). \end{aligned}$$

Proof. Assume that $\langle (\tilde{\alpha}, \tilde{\beta}); X \rangle$ is a double-framed soft deductive system of X over U. Combining (a3) and Definition 3.1, we have

$$\tilde{\alpha}(x-y) \supseteq \tilde{\alpha}((x-y)-x) \cap \tilde{\alpha}(x) = \tilde{\alpha}(0) \cap \tilde{\alpha}(x)$$
$$= \tilde{\alpha}(x),$$

and

$$\tilde{\beta}(x-y) \subseteq \tilde{\beta}((x-y)-x) \cup \tilde{\beta}(x) = \tilde{\beta}(0) \cup \tilde{\beta}(x)$$

$$=\tilde{\beta}(x)$$

for all $x, y \in X$. Since

$$(x - ((x - a) - b)) - a = (x - a) - ((x - a) - b) \le b,$$

it follows from (1) that

$$\tilde{\alpha}((x - ((x - a) - b)) - a) \supseteq \tilde{\alpha}(b),$$

and

$$\tilde{\beta}((x - ((x - a) - b)) - a) \subseteq \tilde{\beta}(b)$$

so from (c2) that

$$\begin{split} \tilde{\alpha}(x - ((x - a)) - b)) \\ &\supseteq \tilde{\alpha}((x - ((x - a) - b)) - a) \cap \tilde{\alpha}(a) \\ &\supseteq \tilde{\alpha}(a) \cap \tilde{\alpha}(b), \end{split}$$

and

$$\begin{split} \tilde{\beta}(x - ((x - a)) - b)) \\ &\subseteq \tilde{\beta}((x - ((x - a) - b)) - a) \cup \tilde{\beta}(a) \\ &\subseteq \tilde{\beta}(a) \cup \tilde{\beta}(b). \end{split}$$

Conversely assume that $\langle (\tilde{\alpha}, \tilde{\beta}); X \rangle$ be a double-framed soft set of X over U satisfying conditions (3) and (4). If we take y = x in (3), then $\tilde{\alpha}(0) = \tilde{\alpha}(x - x) \supseteq \tilde{\alpha}(x)$ and $\tilde{\beta}(0) = \tilde{\beta}(x - x) \subseteq \tilde{\beta}(x)$ for all $x \in X$. By (4), we obtain

$$\begin{split} \tilde{\alpha}(x) &= \tilde{\alpha}(x-0) \\ &= \tilde{\alpha}(x - ((x-y) - (x-y))) \\ &= \tilde{\alpha}(x - ((x - (x-y)) - y)) \\ &\supseteq \tilde{\alpha}(x-y) \cap \tilde{\alpha}(y), \end{split}$$

and

$$\begin{split} \tilde{\beta}(x) &= \tilde{\beta}(x-0) \\ &= \tilde{\beta}(x - ((x-y) - (x-y))) \\ &= \tilde{\beta}(x - ((x-(x-y)) - y)) \\ &\subseteq \tilde{\beta}(x-y) \cup \tilde{\beta}(y) \end{split}$$

for all $x, y \in X$. Hence $\langle (\tilde{\alpha}, \tilde{\beta}); X \rangle$ is a double-framed soft deductive system of X over U.

Proposition 3.8. Every double-framed soft deductive system

 $\left\langle (\tilde{\alpha}, \tilde{\beta}); X \right\rangle$ of X over U satisfies the following assertion:

$$(\forall x, y \in X) \left(\exists x \lor y \Rightarrow \left(\begin{array}{c} \tilde{\alpha}(x \lor y) \supseteq \tilde{\alpha}(x) \cap \tilde{\alpha}(y) \\ \tilde{\beta}(x \lor y) \subseteq \tilde{\beta}(x) \cap \tilde{\beta}(y) \end{array} \right) \right).$$
(5)

Proof. Suppose there exists $x \lor y$ for $x, y \in X$. Let w be an upper bound of x and y. Then $x \lor y = w - ((w - y) - x)$ is the least upper bound for x and y (see Proposition 2.1), and so

$$\tilde{\alpha}(x \lor y) = \tilde{\alpha}(w - ((w - y) - x)) \supseteq \tilde{\alpha}(x) \cap \tilde{\alpha}(y),$$

and

$$\tilde{\beta}(x \lor y) = \tilde{\beta}(w - ((w - y) - x)) \subseteq \tilde{\beta}(x) \cup \tilde{\beta}(y)$$

by (4). This completes the proof.

Proposition 3.9. Let $\langle (\tilde{\alpha}, \tilde{\beta}); X \rangle$ be a double-framed soft of X over U. Then $\langle (\tilde{\alpha}, \tilde{\beta}); X \rangle$ is a double-framed soft deductive system set of X over U if and only if it satisfies:

$$(\forall x, y, z \in X) (x - y \le z \Rightarrow \begin{pmatrix} \tilde{\alpha}(x) \supseteq \tilde{\alpha}(y) \cap \tilde{\alpha}(z), \\ \tilde{\beta}(x) \subseteq \tilde{\beta}(y) \cup \tilde{\beta}(z) \end{pmatrix}).$$
(6)

Proof. Assume that $\langle (\tilde{\alpha}, \tilde{\beta}); X \rangle$ is a double-framed soft of X over U and let $x, y, z \in X$ be such that $x - y \leq z$. Then $\tilde{\alpha}(z) \subseteq \tilde{\alpha}(x-y)$ and $\tilde{\beta}(z) \supseteq \tilde{\beta}(x-y)$ by (1). It follows from (c2) that

$$\tilde{\alpha}(x) \supseteq \tilde{\alpha}(x-y) \cap \tilde{\alpha}(y) \supseteq \tilde{\alpha}(y) \cap \tilde{\alpha}(z),$$

and

$$\tilde{\beta}(x) \subseteq \tilde{\beta}(x-y) \cup \tilde{\beta}(y) \subseteq \tilde{\beta}(y) \cup \tilde{\beta}(z)$$

Conversely suppose that $\left\langle (\tilde{\alpha}, \tilde{\beta}); X \right\rangle$ satisfies (6). Since $0 - y \leq y$ for all $y \in X$, we have

$$\tilde{\alpha}(0) \supseteq \tilde{\alpha}(y) \cap \tilde{\alpha}(y) = \tilde{\alpha}(y),$$

and

$$\tilde{\beta}(0) \subseteq \tilde{\beta}(y) \cup \tilde{\beta}(y) = \tilde{\beta}(y)$$

by (6). Thus (c1) is valid. Since $x - (x - y) \le y$ for all $x, y \in X$ by (a4), it follows from (6) that $\tilde{\alpha}(x) \supseteq \tilde{\alpha}(x - y) \cap \tilde{\alpha}(y)$ and $\tilde{\beta}(x) \subseteq \tilde{\beta}(x - y) \cup \tilde{\beta}(y)$. Hence $\langle (\tilde{\alpha}, \tilde{\beta}); X \rangle$ is a double-framed soft deductive system of X over U. \Box

Let
$$\left\langle (\tilde{\alpha}, \tilde{\beta}); A \right\rangle$$
 and $\left\langle (\tilde{f}, \tilde{g}); B \right\rangle$ be double-framed soft sets

over a common universe U. Then $\langle (\tilde{\alpha}, \tilde{\beta}); A \rangle$ is called a *double*framed soft subset [10] of $\langle (\tilde{f}, \tilde{g}); B \rangle$, denoted by

$$\left\langle (\tilde{\alpha},\tilde{\beta});A\right\rangle \tilde{\subseteq}\left\langle (\tilde{f},\tilde{g});B\right\rangle ,$$

if (i) $A \subseteq B$,

(ii) $(\forall e \in A)$ $\begin{pmatrix} \tilde{\alpha}(e) \text{ and } \tilde{f}(e) \text{ are identical approximations,} \\ \tilde{\beta}(e) \text{ and } \tilde{g}(e) \text{ are identical approximations.} \end{pmatrix}$

Theorem 3.10. Let $\langle (\tilde{\alpha}, \tilde{\beta}); A \rangle$ be a double-framed soft subset of a double-framed soft set $\langle (\tilde{f}, \tilde{g}); B \rangle$. If $\langle (\tilde{f}, \tilde{g}); B \rangle$ is a double-framed soft deductive system of *B* over *U*, then so is $\langle (\tilde{\alpha}, \tilde{\beta}); A \rangle$.

Proof. Since $x \in A$ for all $x \in B$, we have $\tilde{\alpha}(x) = \tilde{f}(x) \subseteq \tilde{f}(0) = \tilde{\alpha}(0)$ and $\tilde{\beta}(x) = \tilde{g}(x) \supseteq \tilde{g}(0) = \tilde{\beta}(0)$. Let $x, y \in A$. Then $x, y \in B$, and so

$$\begin{split} \tilde{\alpha}(x-y) \cap \tilde{\alpha}(y) &= \tilde{f}(x-y) \cap \tilde{f}(y) \subseteq \tilde{f}(x) = \tilde{\alpha}(x), \\ \tilde{\beta}(x-y) \cup \tilde{\beta}(y) &= \tilde{g}(x-y) \cup \tilde{g}(y) \supseteq \tilde{g}(x) = \tilde{\beta}(x). \end{split}$$

Hence $\left\langle (\tilde{\alpha}, \tilde{\beta}); A \right\rangle$ is a double-framed soft deductive system of A over U.

Theorem 3.11. Let $\langle (\tilde{\alpha}, \tilde{\beta}); X \rangle$ be a double-framed soft of X over U. Then the set

$$\mathcal{D}(0) := \{ x \in X \mid \tilde{\alpha}(x) = \tilde{\alpha}(0), \tilde{\beta}(x) = \tilde{\beta}(0) \}$$

is a deductive system of X.

Proof. Obviously $0 \in \mathcal{D}(0)$. Let $x, y \in X$ be such that $x - y \in \mathcal{D}(0)$ and $y \in \mathcal{D}(0)$. Then $\tilde{\alpha}(x - y) = \tilde{\alpha}(0) = \tilde{\alpha}(y)$ and $\tilde{\beta}(x - y) = \tilde{\beta}(0) = \tilde{\beta}(y)$. This implies that $\tilde{\alpha}(x) \supseteq \tilde{\alpha}(x - y) \cap \tilde{\alpha}(y) = \tilde{\alpha}(0)$ and $\tilde{\beta}(x) \subseteq \tilde{\beta}(x - y) \cup \tilde{\beta}(y) = \tilde{\beta}(0)$ by (c2). Since $\tilde{\alpha}(0) \supseteq \tilde{\alpha}(x)$ and $\tilde{\beta}(0) \subseteq \tilde{\beta}(x)$ for all $x \in X$, it follows that $\tilde{\alpha}(x) = \tilde{\alpha}(0)$ and $\tilde{\beta}(x) = \tilde{\beta}(0)$ so that $x \in \mathcal{D}(0)$. Therefore $\mathcal{D}(0)$ is a deductive system of X.

For two double-framed soft sets $\langle (\tilde{\alpha}, \tilde{\beta}); A \rangle$ and $\langle (\tilde{f}, \tilde{g}); A \rangle$ over U, the *double-framed soft int-uni set* [10] of $\langle (\tilde{\alpha}, \tilde{\beta}); A \rangle$ and $\langle (\tilde{f}, \tilde{g}); A \rangle$ is defined to be a double-framed soft set

$$\left\langle (ilde{lpha}\otimes ilde{f}, ilde{eta}\oplus ilde{g});A
ight
angle ,$$

where

$$\begin{split} \tilde{\alpha} \otimes \tilde{f} : A \to \mathscr{P}(U), \; x \mapsto \tilde{\alpha}(x) \cap \tilde{f}(x) \\ \tilde{\beta} \oplus \tilde{g} : A \to \mathscr{P}(U), \; x \mapsto \tilde{\beta}(x) \cup \tilde{g}(x). \end{split}$$

It is denoted by $\left\langle (\tilde{\alpha}, \tilde{\beta}); A \right\rangle \sqcap \left\langle (\tilde{f}, \tilde{g}); A \right\rangle = \left\langle (\tilde{\alpha} \otimes \tilde{f}, \tilde{\beta} \oplus \tilde{g}); A \right\rangle.$

Theorem 3.12. The double-framed soft int-uni set of two double-framed soft deductive system $\langle (\tilde{\alpha}, \tilde{\beta}); A \rangle$ and $\langle (\tilde{f}, \tilde{g}; A \rangle$ of A over U is a double-framed soft deductive system of A over U.

Proof. For all $x \in A$, we have $(\tilde{\alpha} \otimes \tilde{f})(x) = \tilde{\alpha}(x) \cap \tilde{f}(x) \subseteq \tilde{\alpha}(0) \cap \tilde{f}(0) = (\tilde{\alpha} \otimes \tilde{f})(0)$ and $(\tilde{\beta} \oplus \tilde{g})(0) = \tilde{\beta}(0) \cup \tilde{g}(0) \subseteq \tilde{\beta}(x) \cup \tilde{g}(x) = (\tilde{\beta} \oplus \tilde{g})(x)$. Let $x, y \in A$. Then $x, y \in B$, and so

$$\begin{split} &(\tilde{\alpha}\otimes\tilde{f})(x-y)\cap(\tilde{\alpha}\otimes\tilde{f})(y)\\ &=(\tilde{\alpha}(x-y)\cap\tilde{f}(x-y))\cap(\tilde{\alpha}(y)\cap\tilde{f}(y))\\ &=\tilde{\alpha}(x-y)\cap\tilde{\alpha}(y))\cap(\tilde{f}(x-y)\cap\tilde{f}(y))\\ &\subseteq\tilde{\alpha}(x)\cap\tilde{f}(x)=(\tilde{\alpha}\otimes\tilde{f})(x), \end{split}$$

and

$$\begin{split} &(\hat{\beta} \oplus \tilde{g})(x-y) \cup (\hat{\beta} \oplus \tilde{g})(y) \\ &= (\tilde{\beta}(x-y) \cup \tilde{g}(x-y)) \cup (\tilde{\beta}(y) \cup \tilde{g}(y)) \\ &= (\tilde{\beta}(x-y) \cup \tilde{\beta}(y)) \cup (\tilde{g}(x-y) \cup \tilde{g}(y)) \\ &\supseteq \tilde{\beta}(x) \cup \tilde{g}(x) = (\tilde{\beta} \oplus \tilde{g})(x), \end{split}$$

Hence $\left\langle (\tilde{\alpha} \otimes \tilde{f}, \tilde{\beta} \oplus \tilde{g}); A \right\rangle$ is a double-framed soft deductive system of A over U.

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

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