On the Left and Right Almost Hyperideals of LA-Semihypergroups

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Abstract

In this paper, we define left almost hyperideals, right almost hyperideals, almost hyperideals, and minimal almost hyperideals. We demonstrate that the intersection of almost hyperideals is not required to be an almost hyperideal, but the union of almost hyperideals is an almost hyperideal, which is completely different from the classical algebraic concept of the ideal theory.

Keywords: LA-semihypergroups, Left almost hyperideals, Right almost hyperideals, Almost hyperideals, Minimal almost hyperideals

1. Introduction

Hyperstructures were introduced in 1934 when Marty [1] defined hypergroups, began to study their properties, and applied them to groups. Several papers and books have been written on hyperstructure theory [2,3]. Currently, a book published on hyperstructures [4] describes its applications in rough set theory, cryptography, automata, code automata, probability, geometry, lattices, binary relations, graphs, and hypergraphs.

Kazim and Naseeruddin [5] introduced the study of left almost semigroups (LA-semigroups). They generalized some useful sequels of semigroup theory. Subsequently, Mushtaq and his colleagues [6–13] further described the structure and included many useful results of theory of an LA-semigroup [7,14–20]. An LA-semigroup is the midway structure between a commutative semigroup and a groupoid. Moreover, it has many interesting properties, which are frequently used in commutative and associative algebraic structures.

Hila and Dine [21] presented the concept of LA-semihypergroups. Their paper created a new research direction in non-associative hyperstructures. Yaqoob et al. [22] extended the work of Hila and Dine [21] and characterized intra-regular LA-semihypergroups by their hyperideals using pure left identities. Subsequently, many researchers studied the structures of LA-semihypergroups from different perspectives [23–30].

Azhar and his colleagues [31–34] developed the structures of partially ordered LA-semihypergroups. Nawaz and his colleagues [35–41] developed and studied many non-associative structures. The concept of almost ideals in semigroups was provided by Grosek and Satko [42,43] in 1980 and 1981, respectively. Various other researchers studied almost ideals for different algebraic structures [44–46].

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Recently, for the first time, Suebsung et. al [48] developed the concept of almost hyperideals in semihypergroups (see also [48,49]).

Thus, inspired by the concept by Suebsung et. al [47], we developed structures of almost hyperideals in LA-semihypergroups. These new types of almost hyperideals have two main characteristics. First, they are purely non-associative substructures of LA-semihypergroups. Second, in the construction of these hyperideals, we ensured that a hyperoperation did not yield a single element as in the scenario of the paper by Suebsung et. al [47]. We proved some useful results related to almost hyperideals.

2. LA-Semihypergroups

In this section, we recall some basic concepts from the literature on ideals and LA-semihypergroups that were used in the further development of this article.

Definition 1. A map $\circ : X \times X \longrightarrow \mathcal{P}^*(X)$ is called an *hyperoperation* or *join operation* on the set X , where X is a non-empty set and $\mathcal{P}^*(X) = \mathcal{P}(X) \setminus \{\emptyset\}$ indicates the all non-empty subsets of X . A hypergroupoid is a set (X) for which the binary operation is a hyperoperation.

If A and B are two non-empty subsets of X , then we express the product as follows:

$$A \circ B = \bigcup_{x_1 \in A, x_2 \in B} x_1 \circ x_2, \quad x_1 \circ A = \{x_1\} \circ A$$

and $x_1 \circ B = \{x_1\} \circ B$.

Definition 2 ([21,22]). A hypergroupoid $(X, \circ)$ is called an LA-semihypergroup if for $x_1, x_2, x_3 \in X$,

$$(x_1 \circ x_2) \circ x_3 = (x_3 \circ x_2) \circ x_1.$$

The law $(x_1 \circ x_2) \circ x_3 = (x_3 \circ x_2) \circ x_1$ is known as left invertive law.

Every LA-semihypergroup satisfies the law

$$(x_1 \circ x_2) \circ (x_3 \circ x_4) = (x_1 \circ x_3) \circ (x_2 \circ x_4)$$

for all $x_1, x_2, x_3, x_4 \in X$. This law is known as medial law (cf. [21]).

Definition 3. [22] Let X be an LA-semihypergroup. Element $e \in X$ is called a

1. Left identity (resp. pure left identity) if for all $x_1 \in X$, $x_1 \in e \circ x_1$ (resp. $x_1 = e \circ x_1$),
2. Right identity (resp. pure right identity) if for all $x_1 \in X$, $x_1 \in x_1 \circ e$ (resp. $x_1 = x_1 \circ e$),
3. Identity (resp. pure identity) if for all $z_1 \in H$, $z_1 \in e \circ z_1 \cap z_1 \circ e$ (resp. $x_1 = e \circ z_1 \cap z_1 \circ e$).

Example 1 ([22]). Let $X = \{l_1, l_2, l_3, l_4\}$ with the binary hyperoperation defined as follows:

$\circ$	l_1	l_2	l_3	l_4
l_1	l_1	l_2	l_3	l_4
l_2	l_3	$\{l_2, l_3\}$	$\{l_2, l_3\}$	l_4
l_3	l_2	$\{l_2, l_3\}$	$\{l_2, l_3\}$	l_4
l_4	l_4	l_4	l_4	H

Hence X is an LA-semihypergroup because it satisfies the left invertive law. AS the table shows, l_1 is a left identity of X .

Lemma 1 ([22]). Let X be an LA-semihypergroup with pure left identity e ; then $x_1 \circ (x_2 \circ x_3) = x_2 \circ (x_1 \circ x_3)$ holds for all $x_1, x_2, x_3 \in X$.

Lemma 2 ([22]). Let X be an LA-semihypergroup with pure left identity e ; then $(x_1 \circ x_2) \circ (x_3 \circ x_4) = (x_4 \circ x_2) \circ (x_3 \circ x_1)$ holds for all $x_1, x_2, x_3, x_4 \in X$.

The law $(x_1 \circ x_2) \circ (x_3 \circ x_4) = (x_4 \circ x_2) \circ (x_3 \circ x_1)$ is called a paramedial law.

Definition 4 ([47]). Let $(H, \circ)$ be a semihypergroup.

(1) A non-empty subset L of H is called the left almost hyperideal of H if

$$x \circ L \cap L \neq \phi, \forall x \in H.$$

(2) A non-empty subset L of H is called the right almost hyperideal of H if

$$L \circ x \cap L \neq \phi, \forall x \in H.$$

(3) A non-empty subset L of H is called the almost hyperideal of H if L is both a left and right almost hyperideal of H .

3. Almost Hyperideals

Here we define left almost hyperideals, right almost hyperideals, almost hyperideals, minimal almost hyperideals, and some interesting properties.

Definition 5. Let $(H, *)$ be an LA-semihypergroup.

(1) A non-empty subset L of H is called the left almost hyperideal of H if

$$x * L \cap L \neq \phi, \forall x \in H.$$

(2) A non-empty subset R of H is called the right almost hyperideal of H if

$$R * x \cap R \neq \phi, \forall x \in H.$$

(3) A non-empty subset I of H is called the almost hyperideal of H if I is both a left and right almost hyperideal of H .

Example 2. Let $H = \{e, l, m\}$ with the binary operation $*$ defined as follows:

*	e	l	m
e	e	l	m
l	m	$\{l, m\}$	m
m	l	l	$\{l, m\}$

$(H, *)$ is an LA-semihypergroup. Now, if $L = \{e, l\} \subseteq H$ with

*	e	l
e	e	l
l	m	$\{l, m\}$

we observe that $(L, *)$ is a left almost hyperideal of $(H, *)$, but it is not a right almost hyperideal as $\{e, l\} * m \cap \{e, l\} = \phi$.

Remark 1. The structure of left almost hyperideals is not LA-subsemihypergroups as in Example 2, where $L = \{e, l\} \subseteq H$ is not closed under the same binary operation as H .

Example 3. Consider an LA-semihypergroup $H = \{a, b, c\}$ under the hyperoperation defined by

$\circ$	a	b	c
a	a	b	c
b	c	$\{b, c\}$	$\{b, c\}$
c	b	$\{b, c\}$	$\{b, c\}$

If $R = \{b, c\} \subseteq H$ with the same binary operation $\circ$ that is defined in H , we can observe that R is a right almost hyperideal of H .

Lemma 3. Let $(H, *)$ be a LA-semihypergroup.

(1) Every left hyperideal of H is a left almost hyperideal of H .

(2) Every right hyperideal of H is a left almost hyperideal of H .

(3) Every hyperideal of H is an almost hyperideal of H .

Proof. (1) Assume that L is a left hyperideal of H . Let $x \in H$. Then, $x * L \subseteq L$. Therefore, $x * L \cap L \neq \phi$.

(2) Let R be a right hyperideal of H . Let $y \in H$. Then, $R * y \subseteq R$. Therefore, $R * y \cap R \neq \phi$.

(3) Follows from (1) and (2) $\square$

Remark 2. The converse of Lemma 3 may or may not true generally as in the following Example 4.

Example 4. Let $H = \{x, y, z\}$ be a finite set with hyperoperation $*$ defined as follows:

*	x	y	z
x	x	x	x
y	x	H	z
z	x	H	H

Thus, $(H, *)$ is an LA-semihypergroup. Let $L = \{x, y\}$ under the same binary operation $*$, i.e.,

*	x	y
x	x	x
y	x	H

Here, L is an almost hyperideal of H , but it is not a hyperideal of H .

Theorem 1. Let $(H, *)$ be an LA-semihypergroup.

(1) If L_1 and L_2 are two left almost hyperideals of H , then $L_1 \cup L_2$ is also a left almost hyperideal.

(2) If R_1 and R_2 are two right almost hyperideals of H , then $R_1 \cup R_2$ is also a right almost hyperideal.

(3) If I_1 and I_2 are two almost hyperideals of H , then $I_1 \cup I_2$ is also an almost hyperideal.

Proof. Same as provided by Suebsung et. al [47]. $\square$

Remark 3. (1) If L_1 and L_2 are two left almost hyperideals of H , then $L_1 \cap L_2$ may or may not a left almost hyperideal.

(2) If R_1 and R_2 are two right almost hyperideals of H , then $R_1 \cap R_2$ may or may not a right almost hyperideal.

(3) If I_1 and I_2 are two almost hyperideals of H , then $I_1 \cap I_2$ may or may not left almost hyperideal.

Example 5. Let $H = \{1, 2, 3, 4, 5, 6\}$ be a non-empty set with hyperoperation $*$, defined as follows: then $(H, *)$ is an LA-

*	1	2	3	4	5	6
1	{1, 2, 3, 4}	{2, 3, 4, 5, 6}	{4, 5, 6}	{2, 3, 4, 5, 6}	{2, 3, 4, 5, 6}	{2, 3, 4, 5, 6}
2	{1, 3, 4, 5, 6}	{1, 2, 3, 4, 5, 6}	{1, 3, 4, 5, 6}	{1, 3, 6}	{1, 5, 6}	{1, 3, 4, 5, 6}
3	{1, 2, 4, 5, 6}	{1, 2}	{1, 2, 3, 5, 6}	{1, 2, 4, 5, 6}	{1, 2, 6}	{1, 2, 4, 5, 6}
4	{1, 5, 6}	{1, 2, 6}	{1, 2, 5, 6}	{1, 2, 5, 6}	{1, 2, 5, 6}	{1, 2, 5, 6}
5	{1, 2, 3, 4, 6}	{1, 2, 3, 4, 6}	{1, 2, 3, 4, 6}	{1, 3, 4, 6}	{1, 2, 3, 4, 5, 6}	{1, 3, 4, 6}
6	{1, 2, 3, 4, 5}	{1, 4, 5}	{1, 2, 3, 4, 5}	{1, 4, 5}	{1, 2, 3, 4, 5}	{1, 2, 3, 4, 5}

semihypergroup. Let $L_1 = \{1, 2, 3, 4\}$ and $L_2 = \{3, 4, 5, 6\}$ be two left almost hyperideals of H . Thus, $L_1 \cap L_2 = \{3, 4\}$ is not a left almost hyperideal of H , as we can observe that $4 * \{3, 4\} \cap \{3, 4\} = \phi$. However, $L_1 \cup L_2 = \{1, 2, 3, 4, 5, 6\}$ is a left almost hyperideal of H .

Lemma 4. If L is a left almost hyperideal of an LA-semihypergroup H with a left identity e ; then, aL is a left almost hyperideal of H .

Proof. However, if aL is not a left almost hyperideal, i.e.,

$$\begin{aligned} x * aL \cap aL &= \phi \text{ for all } x \in H \\ &\Rightarrow a * xL \cap aL \\ &= \phi, \text{ by } (ab)c = b(ac) \\ &\Rightarrow xL \cap L = \phi, \end{aligned}$$

this is a contradiction to the given condition. Hence, $x * aL \cap aL \neq \phi$ and aL is a left almost hyperideal of H . $\square$

Lemma 5. If L is a left almost hyperideal of an LA-semihypergroup H with a left identity e , then L^2 is a left almost hyperideal of H .

Proof. However, if L^2 is not a left almost hyperideal, i.e.,

$$\begin{aligned} x * L^2 \cap L^2 &= \phi \text{ for all } x \in H, \\ &\Rightarrow x * L * L \cap L * L \\ &\Rightarrow (x * L) * L \cap L * L \\ &\Rightarrow L * (x * L) \cap L * L = \phi, \text{ by } (ab)c = b(ac) \\ &\Rightarrow xL \cap L = \phi, \end{aligned}$$

this is a contradiction to the given condition. Hence, $x * L^2 \cap L^2 \neq \phi$ and L^2 is a left almost hyperideal of H . $\square$

Lemma 6. If I is a left almost hyperideal of H , then I^2 is an almost hyperideal of H .

Proof. First, we must demonstrate that I^2 is a left-almost hyperideal of H . This, we demonstrate that $x * I^2 \cap I^2 \neq \phi \Rightarrow x * I * I \cap I * I$, as $x * I \cap I \neq \phi$. Hence $x * I^2 \cap I^2 \neq \phi$ Therefore,

I^2 is a left almost ideal of H . Next, we must demonstrate that $I^2 * x \cap I^2 \neq \phi \Rightarrow (I * I) * x \cap I * I$, as $(x * I) * I \cap I * I \neq \phi$ by $(ab)c = (cb)a$. As $x * I \cap I \neq \phi$, $I^2 * x \cap I^2 \neq \phi$. Therefore, I^2 is a right almost hyperideal of H . Hence, I^2 is an almost hyperideal of H . $\square$

Definition 6. An almost hyperideal I of an LA-semihypergroup H is minimal if it does not contain any almost hyperideal of H other than itself.

Example 6. Let $H = \{x_1, x_2, x_3, x_4\}$ with the binary hyperoperation defined as follows:

$\circ$	x_1	x_2	x_3	x_4
x_1	x_1	x_2	x_3	x_4
x_2	x_3	$\{x_2, x_3\}$	$\{x_2, x_3\}$	x_4
x_3	x_2	$\{x_2, x_3\}$	$\{x_2, x_3\}$	x_4
x_4	x_4	x_4	x_4	H

Hence H is an LA-semihypergroup. Let $I_1 = \{x_1, x_2, x_4\}$, $I_2 = \{x_1, x_3, x_4\}$ and $I_3 = \{x_4\}$ are the almost hyperideals of H . Thus, I_3 is a minimal almost hyperideal of H .

Theorem 2. Let H be an LA-semihypergroup. If L is a minimal almost left hyperideal of H , then $a * L$ is also a minimal almost left hyperideal of H for every weak idempotent a .

Proof. It is known that $a * L$ is a left almost hyperideal of H . Next, we must demonstrate that $a * L$ is minimal almost left hyperideal of H . Let L be a left almost hyperideal of H , which is properly contained in $a * L$. We define $K = \{i \in I : a * i \subseteq L\}$ and let $y \in K$. Let $a * i \in L \Rightarrow x * (a * i) \cap a * i \neq \phi \Rightarrow x * L \cap L \neq \phi \Rightarrow a * i \in L$, which is contradiction to the minimality of L . Therefore, $a * L$ is minimal almost left hyperideal. $\square$

4. Conclusion

In this paper, we have introduced new types of hyperideals in non-associative structures of semihypergroups and indicated some interesting properties. In the future, we aim to obtain more properties of the proposed ideal theory.

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

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