Intuitionistic Fuzzy Ideals of Ternary Near-Rings

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Abstract

We define the concept of intuitionistic fuzzy ideals of ternary near-rings as a generalization of fuzzy ideals, and we investigate some of their properties. Moreover, we characterize the notions of Noetherian and Artinian ternary near-rings using their intuitionistic fuzzy ideals.

Keywords: Ternary near-ring, Fuzzy ideal, Intuitionistic fuzzy ideal, Intuitionistic fuzzy set

1. Introduction

The concept of fuzzy sets was first introduced by Zadeh [1] as a function of a nonempty set X on the unit interval $[0, 1]$. The first inspired application to many algebraic structures was the concept of fuzzy groups, introduced by Rosenfeld [2]. Lui [3] studied the fuzzy ideals of rings, and many researchers [4–6] have extended these concepts. The notions of fuzzy subnear-rings and fuzzy left (resp. right) ideals in near-rings were introduced by Abou-Zaid in [7]. They have been studied by many authors [8–10].

The concept of intuitionistic fuzzy sets was introduced by Atanassov [11–13] as a generalization of the concept of fuzzy sets. Fuzzy sets give the degree of membership of an element in a given set. Intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership. This theory has been studied by many mathematicians [14–19]. Biswas [20] considered the notion of intuitionistic fuzzy subgroups of groups. In [21], the authors presented the concept of intuitionistic fuzzy ideals of semi-rings. Later, the concept of intuitionistic fuzzy ideals of near-rings was introduced and studied by Zhan and Ma [22].

In 2012, Nakkhasen and Pibajomme [23] introduced the concept of left ternary near-rings and investigated some properties of L -fuzzy ideals of ternary near-rings, where L is a complete lattice with the greatest element 1 and the least element 0. Later, Uma Maheswari and Meera [24, 25] studied the concepts of fuzzy soft ideals and fuzzy soft prime ideals over right ternary near-rings. In this paper, we introduce the notion of intuitionistic fuzzy ideals of ternary near-rings and investigate some of their properties. Also, we introduce and characterize the notions of Noetherian and Artinian ternary near-rings using their intuitionistic fuzzy ideals.

2. Preliminaries

In this section, we present the basic definitions that are used in the following sections of this paper.

Definition 1 [26]. A *ternary semi-group* is an algebraic structure $(N, +, [\])$ such that N is

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a nonempty set, and $[\] : N^3 \rightarrow N$ is a ternary operation satisfying the following associative law: $[[abc]de] = [a[bcd]e] = [ab[cde]]$, for all $a, b, c, d, e \in N$.

Definition 2 [26]. Let A, B , and C be nonempty subsets of a ternary near-ring N . Then, $[ABC] = \{[abc] \in N \mid a \in A, b \in B, c \in C\}$.

Definition 3 [23]. Let N be a nonempty set together with a binary operation $+$ and a ternary operation $[\] : N^3 \rightarrow N$. Then, $(N, +, [\])$ is called a *left ternary near-ring* if it satisfies the following conditions:

- (i) $(N, +)$ is a group (not necessarily abelian);
- (ii) $(N, [\])$ is a ternary semi-group;
- (iii) $[ab(c + d)] = [abc] + [abd]$, for every $a, b, c, d \in N$.

Right ternary near-rings and *lateral ternary near-rings* are defined in a similar manner. In this paper, we focus on left ternary near-rings, and we will use the word “ternary near-rings” to mean “left ternary near-rings.”

Definition 4 [23]. A nonempty subset T of a ternary near-ring N is said to be a *ternary subnear-ring* of N if $a - b \in T$ and $[abc] \in T$, for all $a, b, c \in T$.

Definition 5 [24]. Let N be a nonempty near-ring. Let I be a normal subgroup of $(N, +)$. Then, for every $a, b, c \in N$ and $i \in I$,

- (i) I is called a *left ideal* of N if $[NNI] \subseteq I$;
- (ii) I is called a *right ideal* of N if $[(a + i)bc] - [abc] \in I$;
- (iii) I is called a *lateral ideal* of N if $[a(b + i)c] - [abc] \in I$.

We call I an *ideal* of N if it is a left ideal, a right ideal, and a lateral ideal of N .

Example 1 [23]. Let $N = \{a, b, c, d\}$ be a set with a binary operation $+$ on N as follows:

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

The ternary operation $[\]$ on N is defined by $[xyz] = z$ for all $x, y, z \in N$. Then, we have that $(N, +, [\])$ is a ternary near-ring. Let $I = \{a, b\}$. It follows that I is a ternary subnear-ring of N . Next, we show that I is a left ideal, a right ideal, and a lateral ideal of N , that is, I is an ideal of N .

Definition 6 [23]. Let N and R be ternary near-rings. A mapping $\varphi : N \rightarrow R$ is called a *homomorphism* if $\varphi(a + b) = \varphi(a) + \varphi(b)$ and $\varphi([abc]) = [\varphi(a)\varphi(b)\varphi(c)]$, for all $a, b, c \in N$.

Let X be a nonempty set. A *fuzzy set* [1] of X is a mapping $\mu : X \rightarrow [0, 1]$. Let μ be a fuzzy set of X . The set $U(\mu; t) = \{x \in X \mid \mu(x) \geq t\}$ is called an *upper-level set* of μ , and the set $L(\mu; t) = \{x \in X \mid \mu(x) \leq t\}$ is called a *lower-level set* of μ , where $t \in [0, 1]$. The *complement* of μ denoted by μ^c is the fuzzy set of X defined by $\mu^c(x) = 1 - \mu(x)$, for all $x \in X$. The intersection and union of two fuzzy sets μ and λ of X , denoted by $\mu \cap \lambda$ and $\mu \cup \lambda$, respectively, are defined by letting $x \in X$, $(\mu \cap \lambda)(x) = \min\{\mu(x), \lambda(x)\}$ and $(\mu \cup \lambda)(x) = \max\{\mu(x), \lambda(x)\}$, respectively.

The concept of intuitionistic fuzzy sets was introduced by Atanassov [11–13] as an important generalization of the concept of fuzzy sets. An *intuitionistic fuzzy set* A in a nonempty set X is defined by the form

$$A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\},$$

where $\mu_A : X \rightarrow [0, 1]$ and $\lambda_A : X \rightarrow [0, 1]$ denote the *degree of membership* and the *degree of non-membership* of each $x \in X$ in the set A , and also $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$, for all $x \in X$. For the sake of convenience, we will use the symbol $A = (\mu_A, \lambda_A)$ instead of the intuitionistic fuzzy set $A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in X\}$.

3. Intuitionistic Fuzzy Ideals

In this section, we introduce the concept of intuitionistic fuzzy ideals of ternary near-rings and investigate some of their properties.

Definition 7. An intuitionistic fuzzy set $A = (\mu_A, \lambda_A)$ of a ternary near-ring $(N, +, [\])$ is called an *intuitionistic fuzzy ideal* of N if it satisfies for every $i, x, y, z \in N$,

- (IF1) $\mu_A(x - y) \geq \min\{\mu_A(x), \mu_A(y)\}$;
- (IF2) $\mu_A([xyz]) \geq \min\{\mu_A(x), \mu_A(y), \mu_A(z)\}$;
- (IF3) $\mu_A(y + x - y) \geq \mu_A(x)$;
- (IF4) $\mu_A([xyz]) \geq \mu_A(z)$;
- (IF5) $\mu_A([(x + i)yz] - [xyz]) \geq \mu_A(i)$;
- (IF6) $\mu_A([x(y + i)z] - [xyz]) \geq \mu_A(i)$;
- (AF1) $\lambda_A(x - y) \leq \max\{\lambda_A(x), \lambda_A(y)\}$;

$$(AF2) \quad \lambda_A([xyz]) \leq \max\{\lambda_A(x), \lambda_A(y), \lambda_A(z)\};$$

$$(AF3) \quad \lambda_A(y + x - y) \leq \lambda_A(x);$$

$$(AF4) \quad \lambda_A([xyz]) \leq \lambda_A(z);$$

$$(AF5) \quad \lambda_A([(x + i)yz] - [xyz]) \leq \lambda_A(i);$$

$$(AF6) \quad \lambda_A([x(y + i)z] - [xyz]) \leq \lambda_A(i).$$

Example 2. In Example 1, we define an intuitionistic fuzzy set $A = (\mu_A, \lambda_A)$ of a ternary near-ring N by $\mu_A(c) = \mu_A(d) < \mu_A(b) < \mu_A(a)$ and $\lambda_A(a) < \lambda_A(b) < \lambda_A(c) = \lambda_A(d)$. By routine calculations, it is clear that $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of N .

Proposition 1. If $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of a ternary near-ring N , then $\mu_A(0) \geq \mu_A(x)$ and $\lambda_A(0) \leq \lambda_A(x)$, for all $x \in N$.

Proof. Assume that $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of a ternary near-ring N . Let $x \in N$. Then, $\mu_A(0) = \mu_A(x - x) \geq \min\{\mu_A(x), \mu_A(x)\} = \mu_A(x)$ and $\lambda_A(0) = \lambda_A(x - x) \leq \max\{\lambda_A(x), \lambda_A(x)\} = \lambda_A(x)$. $\square$

Theorem 1. Let N be a ternary near-ring. Then $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of N if and only if for any $t, s \in [0, 1]$, the nonempty sets $U(\mu_A; t)$ and $L(\lambda_A; s)$ are ideals of N .

Proof. Assume that $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of N . Let $s, t \in [0, 1]$. First, let $x, y \in U(\mu_A; t)$. Then, $\mu_A(x) \geq t$ and $\mu_A(y) \geq t$. Thus, $\mu_A(x - y) \geq \min\{\mu_A(x), \mu_A(y)\} \geq t$, and so $x - y \in U(\mu_A; t)$. Second, for any $x \in U(\mu_A; t)$ and $n \in N$, we have $\mu_A(n + x - n) \geq \mu_A(x) \geq t$, and then $n + x - n \in U(\mu_A; t)$. Third, for any $n, m \in N$ and $x \in U(\mu_A; t)$, we have $\mu_A([nm]x) \geq \mu_A(x) \geq t$, that is, $[nm]x \in U(\mu_A; t)$. Finally, for any $i \in U(\mu_A; t)$ and $x, y, z \in N$. Then, $\mu_A([(x + i)yz] - [xyz]) \geq \mu_A(i) \geq t$ and $\mu_A([x(y + i)z] - [xyz]) \geq \mu_A(i) \geq t$. It follows that $[(x + i)yz] - [xyz] \in U(\mu_A; t)$ and $[x(y + i)z] - [xyz] \in U(\mu_A; t)$. Hence, $U(\mu_A; t)$ is an ideal of N . Similarly, we can show that $L(\lambda_A; s)$ is also an ideal of N .

Conversely, assume that for any for any $s, t \in [0, 1]$, the nonempty sets $U(\mu_A; t)$ and $L(\lambda; s)$ are ideals of N . We want to show that μ_A satisfies (IF1)-(IF6) and λ_A satisfies (AF1)-(AF6). Let $i, x, y, z \in N$. (IF1) Let $t_1 \in [0, 1]$ such that $t_1 = \min\{\mu_A(x), \mu_A(y)\}$. Then, $\mu_A(x) \geq t_1$ and $\mu_A(y) \geq t_1$, so $x, y \in U(\mu_A; t_1)$. By assumption, $x - y \in U(\mu_A; t_1)$. Thus, $\mu_A(x - y) \geq t_1 = \min\{\mu_A(x), \mu_A(y)\}$. (IF2) Let $t_2 \in [0, 1]$

such that $t_2 = \min\{\mu_A(x), \mu_A(y), \mu_A(z)\}$. Then, $\mu_A(x) \geq t_2$, $\mu_A(y) \geq t_2$, $\mu_A(z) \geq t_2$, that is, $x, y, z \in U(\mu_A; t_2)$. By assumption, $[xyz] \in U(\mu_A; t_2)$. Thus,

$\mu_A([xyz]) \geq t_2 = \min\{\mu_A(x), \mu_A(y), \mu_A(z)\}$. (IF3) Let $t_3 \in [0, 1]$ such that $t_3 = \mu_A(x)$. Then $x \in U(\mu_A; t_3)$. So, $y + x - y \in U(\mu_A; t_3)$. Hence, $\mu_A(y + x - y) \geq t_3 = \mu_A(x)$. (IF4) Let $t_4 \in [0, 1]$ such that $t_4 = \mu_A(z)$. Then $z \in U(\mu_A; t_4)$. Thus, $[xyz] \in U(\mu_A; t_4)$, and then $\mu_A([xyz]) \geq t_4 = \mu_A(z)$. (IF5) Let $t_5 \in [0, 1]$ such that $t_5 = \mu_A(i)$. Then $i \in U(\mu_A; t_5)$. Thus, $[(x + i)yz] - [xyz] \in U(\mu_A; t_5)$, that is, $\mu_A([(x + i)yz] - [xyz]) \geq t_5 = \mu_A(i)$. (IF6) Let $t_6 \in [0, 1]$ such that $t_6 = \mu_A(i)$. Then, $i \in U(\mu_A; t_6)$. Thus, $[x(y + i)z] - [xyz] \in U(\mu_A; t_6)$. It follows that $\mu_A([x(y + i)z] - [xyz]) \geq t_6 = \mu_A(i)$. Therefore, μ_A satisfies (IF1)-(IF6). Similarly, we can prove that λ_A satisfies (AF1)-(AF6). This completes the proof. $\square$

Example 3. In Example 2, it is clear that $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of N . Then, we can show that the upper-level sets of N are $U(\mu_A; \mu_A(a)) = \{a\}$, $U(\mu_A; \mu_A(b)) = \{a, b\}$, $U(\mu_A; \mu_A(c)) = N$, and $U(\mu_A; \mu_A(d)) = N$. Also, the lower-level sets of N are $L(\lambda_A; \lambda_A(a)) = \{a\}$, $L(\lambda_A; \lambda_A(b)) = \{a, b\}$, $L(\lambda_A; \lambda_A(c)) = N$, and $L(\lambda_A; \lambda_A(d)) = N$. By Theorem 1, it follows that $\{a\}$, $\{a, b\}$, and N are the ideals of N .

Theorem 2. Let N be a ternary near-ring. Then, $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of N if and only if $A^c = (\lambda_A^c, \mu_A^c)$ is an intuitionistic fuzzy ideal of N .

Proof. Assume that $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of N . Let $i, x, y, z \in N$. (IF1) $\lambda_A^c(x - y) = 1 - \lambda_A(x - y) \geq 1 - \max\{\lambda_A(x), \lambda_A(y)\} = \min\{1 - \lambda_A(x), 1 - \lambda_A(y)\} = \min\{\lambda_A^c(x), \lambda_A^c(y)\}$. Similarly, we can show that λ_A^c satisfies (IF2)-(IF6). Next, we consider (AF1) $\mu_A^c(x - y) = 1 - \mu_A(x - y) \leq 1 - \min\{\mu_A(x), \mu_A(y)\} = \max\{1 - \mu_A(x), 1 - \mu_A(y)\} = \max\{\mu_A^c(x), \mu_A^c(y)\}$. Similarly, we can show that (AF2)-(AF6). Hence, $A^c = (\lambda_A^c, \mu_A^c)$ is an intuitionistic fuzzy ideal of N .

Conversely, assume that $A^c = (\lambda_A^c, \mu_A^c)$ is an intuitionistic fuzzy ideal of N . Let $i, x, y, z \in N$. (IF1) $1 - \mu_A(x - y) = \mu_A^c(x - y) \leq \max\{\mu_A^c(x), \mu_A^c(y)\} = \max\{1 - \mu_A(x), 1 - \mu_A(y)\} = 1 - \min\{\mu_A(x), \mu_A(y)\}$. This implies that $\mu_A(x - y) \geq \min\{\mu_A(x), \mu_A(y)\}$. Similarly, we can show that μ_A satisfies (IF2)-(IF6). Next, we consider (AF1) $1 - \lambda_A(x - y) = \lambda_A^c(x - y) \geq \min\{\lambda_A^c(x), \lambda_A^c(y)\} = \min\{1 - \lambda_A(x), 1 - \lambda_A(y)\} = 1 - \max\{\lambda_A(x), \lambda_A(y)\}$. It follows that $\lambda_A(x - y) \leq \max\{\lambda_A(x), \lambda_A(y)\}$. Similarly, we can show that λ_A

satisfies (AF2)-(AF6). Therefore, $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of N . $\square$

The following theorem immediately follows from Theorem 2.

Theorem 3. Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy set of a ternary near-ring N . Then, $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of N if and only if $A^\mu = (\mu_A, \mu_A^c)$ and $A^\lambda = (\lambda_A^c, \lambda_A)$ are intuitionistic fuzzy ideals of N .

Theorem 4. If $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ are intuitionistic fuzzy ideals of a ternary near-ring N , then $A \cup B = (\mu_A \cup \mu_B, \lambda_A \cap \lambda_B)$ is also an intuitionistic fuzzy ideal of N .

Proof. Assume that $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ are intuitionistic fuzzy ideals of a ternary near-ring N . Clearly, $A \cup B$ is an intuitionistic fuzzy set of N . Let $i, x, y, z \in N$. (IF1) Then,

$$\begin{aligned} (\mu_A \cup \mu_B)(x - y) &= \max\{\mu_A(x - y), \mu_B(x - y)\} \\ &\geq \max\{\min\{\mu_A(x), \mu_A(y)\}, \min\{\mu_B(x), \mu_B(y)\}\} \\ &= \min\{\max\{\mu_A(x), \mu_B(x)\}, \max\{\mu_A(y), \mu_B(y)\}\} \\ &= \min\{(\mu_A \cup \mu_B)(x), (\mu_A \cup \mu_B)(y)\}. \end{aligned}$$

The proofs of (IF2)-(IF6) are similar to that of (IF1). Next, (AF1) we have

$$\begin{aligned} (\lambda_A \cap \lambda_B)(x - y) &= \min\{\lambda_A(x - y), \lambda_B(x - y)\} \\ &\leq \min\{\max\{\lambda_A(x), \lambda_A(y)\}, \max\{\lambda_B(x), \lambda_B(y)\}\} \\ &= \max\{\min\{\lambda_A(x), \lambda_B(x)\}, \min\{\lambda_A(y), \lambda_B(y)\}\} \\ &= \max\{(\lambda_A \cap \lambda_B)(x), (\lambda_A \cap \lambda_B)(y)\}. \end{aligned}$$

The proofs of (AF2)-(AF6) are similar to that of (AF1). Therefore, $A \cup B$ is an intuitionistic fuzzy ideal of N . $\square$

Theorem 5. If $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ are intuitionistic fuzzy ideals of a ternary near-ring N , then $A \cap B = (\mu_A \cap \mu_B, \lambda_A \cup \lambda_B)$ is also an intuitionistic fuzzy ideal of N .

Proof. The proof is similar to Theorem 4. $\square$

Let $f : N \rightarrow R$ be a homomorphism of ternary near-rings. For any $A = (\mu_A, \lambda_A)$ of R , we define a new $A^f = (\mu_A^f, \lambda_A^f)$ of N by $\mu_A^f(x) = \mu_A(f(x))$ and $\lambda_A^f(x) = \lambda_A(f(x))$, for all $x \in N$.

Theorem 6. Let $f : N \rightarrow R$ be a homomorphism of ternary near-rings. If $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of R , then $A^f = (\mu_A^f, \lambda_A^f)$ is an intuitionistic fuzzy ideal of N .

Proof. Assume that $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of R . Let $i, x, y, z \in N$. Then, (IF1) $\mu_A^f(x - y) = \mu_A(f(x - y)) = \mu_A(f(x) - f(y)) \geq \min\{\mu_A(f(x)), \mu_A(f(y))\} = \min\{\mu_A^f(x), \mu_A^f(y)\}$. Similarly, we can prove that μ_A^f satisfies (IF2)-(IF6). Now, (AF1) $\lambda_A^f(x - y) = \lambda_A(f(x - y)) = \lambda_A(f(x) - f(y)) \leq \max\{\lambda_A(f(x)), \lambda_A(f(y))\} = \max\{\lambda_A^f(x), \lambda_A^f(y)\}$. Similarly, we can prove that λ_A^f satisfies (AF2)-(AF6). Therefore, $A^f = (\mu_A^f, \lambda_A^f)$ is an intuitionistic fuzzy ideal of N . $\square$

If we strengthen the condition of f , we can construct the converse of Theorem 6 as follows.

Theorem 7. Let $f : N \rightarrow R$ be an epimorphism of ternary near-rings, and let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy set of R . If $A^f = (\mu_A^f, \lambda_A^f)$ is an intuitionistic fuzzy ideal of N , then $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of R .

Proof. Assume that $A^f = (\mu_A^f, \lambda_A^f)$ is an intuitionistic fuzzy ideal of N . Let $i, x, y, z \in R$. Then, there exist $n, a, b, c \in N$ such that $f(i) = n, f(x) = a, f(y) = b$ and $f(z) = c$. Then, (IF1) $\mu_A(x - y) = \mu_A(f(a) - f(b)) = \mu_A(f(a - b)) = \mu_A^f(a - b) \geq \min\{\mu_A^f(a), \mu_A^f(b)\} = \min\{\mu_A(f(a)), \mu_A(f(b))\} = \min\{\mu_A(x), \mu_A(y)\}$. Similarly, we can show that μ_A satisfies (IF2)-(IF6). Now, we consider (AF1) $\lambda_A(x - y) = \lambda_A(f(a) - f(b)) = \lambda_A(f(a - b)) = \lambda_A^f(a - b) \leq \max\{\lambda_A^f(a), \lambda_A^f(b)\} = \max\{\lambda_A(f(a)), \lambda_A(f(b))\} = \max\{\lambda_A(x), \lambda_A(y)\}$. Similarly, we can show that λ_A satisfies (AF2)-(AF6). Hence, $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy ideal of R . $\square$

Let μ be a fuzzy set of a nonempty set X and $t \in [0, 1 - \sup_{x \in X} \mu(x)]$. The mapping $\mu^T : X \rightarrow [0, 1]$ is called a *fuzzy translation* [27] of μ if $\mu^T(x) = \mu(x) + t$, for all $x \in X$.

Theorem 8. Let N be a ternary near-ring, $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy set of N , and $t \in [0, \frac{1}{2}(1 - \sup_{x \in N} \{\mu_A(x) + \lambda_A(x)\})]$. Suppose that μ_A^T and λ_A^T are fuzzy translations of μ_A and λ_A with respect to t , respectively. Then, $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of N if and only if $A^T = (\mu_A^T, \lambda_A^T)$ is an intuitionistic fuzzy ideal of N .

Proof. Assume that $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of N . Let $a \in N$ and $t = \frac{1}{2}(1 - \sup_{x \in N} \{\mu_A(x) + \lambda_A(x)\})$. Then, $\mu_A^T(a) + \lambda_A^T(a) = \mu_A(a) + \lambda_A(a) + 2t = \mu_A(a) + \lambda_A(a) + 1 - \sup_{x \in N} \{\mu_A(x) + \lambda_A(x)\} \leq \mu_A(a) + \lambda_A(a) + 1 - (\mu_A(a) + \lambda_A(a)) = 1$. Thus, $A^T = (\mu_A^T, \lambda_A^T)$ is an intuitionistic fuzzy set of N . Let $i, x, y, z \in N$. We have (IF1) $\mu_A^T(x - y) = \mu_A(x - y) + t \geq \min\{\mu_A(x), \lambda_A(y)\} + t = \min\{\mu_A(x) +$

$t, \lambda_A(y) + t\} = \min\{\mu_A^T(x), \lambda_A^T(y)\}$. It is not difficult to show that μ_A^T satisfies (IF2)-(IF6). In addition, (AF1) $\lambda_A^T(x - y) = \lambda_A(x - y) + t \leq \max\{\lambda_A(x), \lambda_A(y)\} + t = \max\{\lambda_A(x) + t, \lambda_A(y) + t\} = \max\{\lambda_A^T(x), \lambda_A^T(y)\}$. Similarly, λ_A^T satisfies (AF2)-(AF6). Hence, $A^T = (\mu_A^T, \lambda_A^T)$ is an intuitionistic fuzzy ideal of N .

Conversely, assume that $A^T = (\mu_A^T, \lambda_A^T)$ is an intuitionistic fuzzy ideal of N . Let $i, x, y, z \in N$. Then, (IF1) $\mu_A(x - y) + t = \mu_A^T(x - y) \geq \min\{\mu_A^T(x), \mu_A^T(y)\} = \min\{\mu_A(x) + t, \mu_A(y) + t\}$, implies that $\mu_A(x - y) \geq \min\{\mu_A(x), \mu_A(y)\}$ because $t \geq 0$. Similarly, we can prove that μ_A satisfies (IF2)-(IF6). Next, we have (AF1) $\lambda_A(x - y) + t = \lambda_A^T(x - y) \leq \max\{\lambda_A^T(x), \lambda_A^T(y)\} = \max\{\lambda_A(x) + t, \lambda_A(y) + t\}$, and so $\lambda_A(x - y) \leq \max\{\lambda_A(x), \lambda_A(y)\}$, as $t \geq 0$. Similarly, λ_A satisfies (AF2)-(AF6). Therefore, $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of N . $\square$

Let μ be a fuzzy set of a nonempty set X and $m \in [0, 1]$. The mapping $\mu^M : X \rightarrow [0, 1]$ is called a *fuzzy multiplication* [27] of μ if $\mu^M(x) = m\mu(x)$, for all $x \in X$.

Theorem 9. Let N be a ternary near-ring, $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy set of N , and $m \in (0, 1]$. Suppose that μ_A^M and λ_A^M are fuzzy multiplications of μ_A and λ_A , respectively, where $\mu_A^M(x) = m\mu_A(x)$ and $\lambda_A^M(x) = m\lambda_A(x)$, for all $x \in N$. Then, $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of N if and only if $A^M = (\mu_A^M, \lambda_A^M)$ is an intuitionistic fuzzy ideal of N .

Proof. Assume that $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of N . Obviously, $A^M = (\mu_A^M, \lambda_A^M)$ is an intuitionistic fuzzy set of N . Let $i, x, y, z \in N$. Then, (IF1) $\mu_A^M(x - y) = m\mu_A(x - y) \geq m \min\{\mu_A(x), \mu_A(y)\} = \min\{m\mu_A(x), m\mu_A(y)\} = \min\{\mu_A^M(x), \mu_A^M(y)\}$. The proofs of (IF2)-(IF6) are similar to that of (IF1). Next, we have $\lambda_A^M(x - y) = m\lambda_A(x - y) \leq m \max\{\lambda_A(x), \lambda_A(y)\} = \max\{m\lambda_A(x), m\lambda_A(y)\} = \max\{\lambda_A^M(x), \lambda_A^M(y)\}$. In the same way, we have λ_A^M satisfying (AF2)-(AF6). Hence, $A^M = (\mu_A^M, \lambda_A^M)$ is an intuitionistic fuzzy ideal of N .

Conversely, assume that $A^M = (\mu_A^M, \lambda_A^M)$ is an intuitionistic fuzzy ideal of N . Let $i, x, y, z \in N$. Then, (IF1) $m\mu_A(x - y) = \mu_A^M(x - y) \geq \min\{\mu_A^M(x), \mu_A^M(y)\} = m \min\{\mu_A(x), \mu_A(y)\}$. Because $m > 0$, $\mu_A(x - y) \geq \min\{\mu_A(x), \mu_A(y)\}$. Uniformly, we have that μ_A satisfies (IF2)-(IF6). Now, consider (AF1) $m\lambda_A(x - y) = \lambda_A^M(x - y) \leq \max\{\lambda_A^M(x), \lambda_A^M(y)\} = m \max\{\lambda_A(x), \lambda_A(y)\}$. This implies that

$$\lambda_A(x - y) \leq \max\{\lambda_A(x), \lambda_A(y)\},$$

as $m > 0$. Similarly, λ_A satisfies (AF2)-(AF6). Therefore, $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of N . $\square$

Let μ be a fuzzy set of a nonempty set X , $m \in [0, 1]$, and $t \in [0, 1 - \sup_{x \in X} \mu(x)]$. The mapping $\mu^{MT} : X \rightarrow [0, 1]$ is called a *fuzzy magnified translation* [28] of μ if $\mu^{MT}(x) = m\mu(x) + t$, for all $x \in X$. The following theorem immediately follows from Theorem 8 and Theorem 9.

Theorem 10. Let N be a ternary near-ring, $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy set of N , $t \in [0, \frac{1}{2}(1 - \sup_{x \in N} \{\mu_A(x) + \lambda_A(x)\})]$, and $m \in (0, 1]$. Suppose that μ^{MT} and λ^{MT} are fuzzy magnified translations of μ_A and λ_A with respect to t and m , respectively. Then, $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of N if and only if $A^{MT} = (\mu_A^{MT}, \lambda_A^{MT})$ is an intuitionistic fuzzy ideal of N .

4. Noetherian and Artinian Ternary Near-Rings

In this section, we define the notions of Noetherian and Artinian ternary near-rings and characterize Noetherian and Artinian ternary near-rings using their intuitionistic fuzzy ideals.

Definition 8. A ternary near-ring N is called *Noetherian* (resp. *Artinian*) if N satisfies the ascending (resp. descending) chain condition on ideals of N , that is, for any ideals $I_1, I_2, I_3, \dots$ of N , with

$$I_1 \subseteq I_2 \subseteq I_3 \subseteq \dots \subseteq I_i \subseteq \dots \\ (I_1 \supseteq I_2 \supseteq I_3 \supseteq \dots \supseteq I_i \supseteq \dots),$$

there exists $n \in \mathbb{N}$ such that $I_i = I_{i+1}$ for all $i \geq n$.

Theorem 11. If every intuitionistic fuzzy ideal of a ternary near-ring N has a finite image of values, then N is Noetherian.

Proof. Assume that every intuitionistic fuzzy ideal of a ternary near-ring N has the finite image of values. Suppose that N is not Noetherian, so there exists an ascending chain condition on ideals of N , that is, $I_0 \subseteq I_1 \subseteq I_2 \subseteq \dots$. We define the intuitionistic fuzzy set $A = (\mu_A, \lambda_A)$ of N by

$$\mu_A(x) = \begin{cases} \frac{1}{n+1} & \text{if } x \in I_{n+1} - I_n; \\ 1 & \text{if } x \in I_0; \\ 0 & \text{if } x \in S - \bigcup_{n=0}^{\infty} I_n, \end{cases}$$

$$\lambda_A(x) = \begin{cases} \frac{n}{n+1} & \text{if } x \in I_{n+1} - I_n; \\ 0 & \text{if } x \in I_0; \\ 1 & \text{if } x \in S - \bigcup_{n=0}^{\infty} I_n, \end{cases}$$

for all $x \in N$. It is not difficult to show that $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of N . We have a contradiction because $I_0 \subseteq I_1 \subseteq I_2 \subseteq \dots$ is an infinitely ascending chain of ideals of N . $\square$

The proof of the following theorem is similar to that of Theorem 11.

Theorem 12. If every intuitionistic fuzzy ideal of a ternary near-ring N has a finite image of values, then N is Artinian.

Theorem 13. A ternary near-ring N is Noetherian if and only if the set of values of any intuitionistic fuzzy ideal of N is a well-ordered subset of $[0, 1]$.

Proof. Assume that N is Noetherian. Suppose that $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of N , which is not a well-ordered subset of $[0, 1]$. Then, there exists an infinite descending sequence $\{t_n\}_{n=1}^{\infty}$ such that $\mu_A(x) = t_n$ and $\lambda_A(x) \leq 1 - t_n$, for some $x \in N$. We define $I_n = \{x \in N \mid \mu_A(x) \geq t_n\}$ and $J_n = \{x \in N \mid \lambda_A(x) \leq 1 - t_n\}$. By Theorem 1, I_n and J_n are ideals of N , for all $n \in \mathbb{N}$. It follows that $I_1 \subset I_2 \subset I_3 \subset \dots$ and $J_1 \subset J_2 \subset J_3 \subset \dots$ are strictly infinite ascending chains of ideals of N . This contradicts our hypothesis.

Conversely, suppose that N is not Noetherian. Then, there exists a strictly infinite ascending chain of ideals of N , namely, $I_1 \subseteq I_2 \subseteq I_3 \subseteq \dots$. Let $I = \bigcup_{n \in \mathbb{N}} I_n$. It easy to show that I is ideal for N . Next, we define the intuitionistic fuzzy set of $A = (\mu_A, \lambda_A)$ of N by

$$\mu_A(x) = \begin{cases} \frac{1}{m} & \text{if } m = \min\{n \in \mathbb{N} \mid x \in I_n\}; \\ 0 & \text{if } x \notin I; \end{cases}$$

$$\lambda_A(x) = \begin{cases} \frac{m-1}{m+1} & \text{if } m = \min\{n \in \mathbb{N} \mid x \in I_n\}; \\ 1 & \text{if } x \notin I, \end{cases}$$

for all $x \in N$. Obviously, $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of N . Because the chain is not finite, $A = (\mu_A, \lambda_A)$ has an infinite ascending sequence of values. This is a contradiction with the idea that the set of values of the intuitionistic fuzzy ideal is not a well-ordered subset of $[0, 1]$. $\square$

Theorem 14. A ternary near-ring N is both Noetherian and

Artinian if and only if every intuitionistic fuzzy ideal of N has a finite image of values.

Proof. Suppose that $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy ideal of N such that $\text{Im}(\mu_A)$ and $\text{Im}(\lambda_A)$ are infinite. By Theorem 1, $U(\mu_A; t_n)$ and $L(\lambda_A; s_m)$ are ideals of N , for all $m, n \in \mathbb{N}$. Because N is Noetherian and by Theorem 13, $\text{Im}(\mu_A)$ and $\text{Im}(\lambda_A)$ are well-ordered subsets of $[0, 1]$. Then, we can separate them into two cases, as follows.

Case 1: Let $t_1 < t_2 < t_3 < \dots$ be an increasing sequence in $\text{Im}(\mu_A)$ and $s_1 > s_2 > s_3 > \dots$ be a decreasing sequence in $\text{Im}(\lambda_A)$. It follows that $U(\mu_A; t_1) \supset U(\mu_A; t_2) \supset U(\mu_A; t_3) \supset \dots$ and $L(\lambda_A; s_1) \supset L(\lambda_A; s_2) \supset L(\lambda_A; s_3) \supset \dots$ are exactly descending chains of ideals of N . Because N is Artinian, there exist $i, j \in \mathbb{N}$ such that $U(\mu_A; t_i) = U(\mu_A; t_{i+k})$ and $L(\lambda_A; s_j) = L(\lambda_A; s_{j+l})$, for all $k, l \in \mathbb{N}$. It turns out that $t_i = t_{i+k}$ and $s_j = s_{j+l}$, for all $k, l \in \mathbb{N}$. This is a contradiction.

Case 2: Let $t_1 > t_2 > t_3 > \dots$ be a decreasing sequence in $\text{Im}(\mu_A)$ and $s_1 < s_2 < s_3 < \dots$ be an increasing sequence in $\text{Im}(\lambda_A)$. It follows that $U(\mu_A; t_1) \supset U(\mu_A; t_2) \supset U(\mu_A; t_3) \supset \dots$ and $L(\lambda_A; s_1) \supset L(\lambda_A; s_2) \supset L(\lambda_A; s_3) \supset \dots$ are absolutely ascending chains of ideals of N . Because N is Noetherian, there exist $i, j \in \mathbb{N}$ such that $U(\mu_A; t_i) = U(\mu_A; t_{i+k})$ and $L(\lambda_A; s_j) = L(\lambda_A; s_{j+l})$, for all $k, l \in \mathbb{N}$. It follows that $t_i = t_{i+k}$ and $s_j = s_{j+l}$, for all $k, l \in \mathbb{N}$. We have a contradiction.

Conversely, it follows by Theorem 11 and Theorem 12. $\square$

5. Conclusion

We introduced the concept of the intuitionistic fuzzy ideal in ternary near-rings as a generalization of their fuzzy ideals and studied some of their properties. We also presented the notions of Noetherian and Artinian ternary near-rings and characterized some of their properties using their intuitionistic fuzzy ideals. In the future, we would like to investigate some of the basic properties of the concepts of fuzzy quasi-ideals and fuzzy bi-ideals in ternary near-rings. Next, we will study the concepts of intuitionistic fuzzy quasi-ideals and intuitionistic fuzzy bi-ideals in ternary near-rings as generalizations of their fuzzy quasi-ideals and fuzzy bi-ideals, respectively.

Conflict of Interest

No potential conflict of interest relevant to this article is reported.

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