Pairwise Precontinuity in Intuitionistic Smooth Bitopological Spaces

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Abstract

The authors introduced the concepts of intuitionistic smooth bitopological spaces, and obtained some characterizations of intuitionistic fuzzy pairwise continuity and intuitionistic fuzzy pairwise semicontinuity of them in the previous papers. Here, we introduce a new notion of intuitionistic fuzzy pairwise precontinuous mappings in intuitionistic smooth bitopological spaces, and obtain six equivalent conditions for intuitionistic fuzzy pairwise precontinuous mappings. Furthermore, we obtain another equivalent condition for a bijective mapping.

Keywords: Intuitionistic smooth bitopology, Pairwise preopen, Pairwise preclosed, Pairwise precontinuous

1. Introduction and Preliminaries

Chang [1] introduced a new concept called the fuzzy topology in a fuzzy set area. The fuzzy topology is a crisp subfamily of fuzzy sets, and hence, it does not represent the notion of vagueness of a fuzzy set. This would be a shortcoming in the process of fuzzification of topological spaces. To overcome these defects, a new definition of fuzzy topology using the openness was introduced by Šostak [2, 3]. His fuzzy topology was later rephrased as “smooth topology” by Ramadan [4].

Also, Chang’s fuzzy topological spaces were generalized into the concept of fuzzy bitopological spaces with pairing the fuzzy topological spaces by Kandil [5]. His fuzzy bitopological spaces were later generalized into a notion of smooth bitopological spaces by Lee and his colleagues [6, 7] by adding vagueness.

On the other hand, the intuitionistic fuzzy topological spaces were introduced by Çoker and his colleague [8, 9]. They are defined with intuitionistic fuzzy sets introduced by Atanassov [10]. Also, new generalization of a smooth topology was introduced by Mondal and Samanta [11], which is called intuitionistic gradation of openness. Lim et al. [12] defined the term “intuitionistic smooth topology,” which is a slight modification of the Mondal and Samanta’s intuitionistic gradation of openness. Thus, it is different from ours.

In the previous papers [13, 14], we introduced the concepts of intuitionistic smooth bitopological spaces, and obtained some characterizations of intuitionistic fuzzy pairwise continuity and intuitionistic fuzzy pairwise semicontinuity of them.

In this paper, by using the idea of intuitionistic fuzzy pairwise preopen and preclosed sets, we introduce intuitionistic fuzzy pairwise precontinuous mappings in intuitionistic smooth bitopological spaces, and obtain the characterizations for the mappings.

Received: Jul. 7, 2019
 Revised : Aug. 19, 2019
 Accepted: Sep. 20, 2019

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Throughout this paper, I denotes the unit interval $[0, 1]$ of the real line and $I_0 = (0, 1]$. A member μ of I^X is called a *fuzzy set* in X . For any $\mu \in I^X$, μ^c denotes the complement $1 - \mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant mappings on X with value of 0 and 1, respectively.

Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A = (\mu_A, \gamma_A),$$

where the functions $\mu_A : X \rightarrow I$ and $\gamma_A : X \rightarrow I$ denote the degree of membership and the degree of nonmembership, respectively, and $\mu_A + \gamma_A \leq 1$. Obviously, every fuzzy set μ in X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1} - \mu)$. $I(X)$ denotes a family of all intuitionistic fuzzy sets in X and “IF” stands for intuitionistic fuzzy.

All the definitions and notations which are not mentioned in this paper, we refer to [13, 14].

Definition 1.1 ([13]). An *intuitionistic smooth topology* on X is a mapping $\mathcal{T} : I(X) \rightarrow I$ which satisfies the following properties:

- 1) $\mathcal{T}(\underline{0}) = \mathcal{T}(\underline{1}) = 1$.
- 2) $\mathcal{T}(A \cap B) \geq \mathcal{T}(A) \wedge \mathcal{T}(B)$.
- 3) $\mathcal{T}(\bigvee A_i) \geq \bigwedge \mathcal{T}(A_i)$.

The pair $(X, \mathcal{T})$ is called an *intuitionistic smooth topological space*.

Definition 1.2 ([13]). A system $(X, \mathcal{T}_1, \mathcal{T}_2)$ consisting of a set X with two intuitionistic smooth topologies $\mathcal{T}_1$ and $\mathcal{T}_2$ on X is called a *intuitionistic smooth bitopological space* (ISBTS for short). Throughout this paper the indices i, j take the value in $\{1, 2\}$ and $i \neq j$.

Definition 1.3 ([13]). Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from an ISBTS X to an ISBTS Y and $r, s \in I_0$. Then f is said to be *IF pairwise (r, s) -continuous* if the induced mapping $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{U}_1)$ is an IF r -continuous mapping and the induced mapping $f : (X, \mathcal{T}_2) \rightarrow (Y, \mathcal{U}_2)$ is an IF s -continuous mapping.

Definition 1.4 ([13]). Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from an ISBTS X to an ISBTS Y and $r, s \in I_0$. Then f is said to be *IF pairwise (r, s) -semicontinuous* if $f^{-1}(A)$ is an IF $(\mathcal{T}_1, \mathcal{T}_2)$ -(r, s)-semiopen set in X for each IF $\mathcal{U}_1$ - r -open set A in Y and $f^{-1}(B)$ is an IF $(\mathcal{T}_2, \mathcal{T}_1)$ -(s, r)-semiopen set in X for each IF $\mathcal{U}_2$ - s -open set B in Y .

Definition 1.5 ([14]). Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from an ISBTS X to an ISBTS Y and $r, s \in I_0$. Then f is said to be

- 1) *IF pairwise (r, s) -open* if the induced mapping $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{U}_1)$ is an IF r -open mapping and the induced mapping $f : (X, \mathcal{T}_2) \rightarrow (Y, \mathcal{U}_2)$ is an IF s -open mapping,
- 2) *IF pairwise (r, s) -closed* if the induced mapping $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{U}_1)$ is an IF r -closed mapping and the induced mapping $f : (X, \mathcal{T}_2) \rightarrow (Y, \mathcal{U}_2)$ is an IF s -closed mapping.

Definition 1.6 ([14]). Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from an ISBTS X to an ISBTS Y and $r, s \in I_0$. Then f is said to be

- 1) *IF pairwise (r, s) -semiopen* if $f(A)$ is IF $(\mathcal{U}_1, \mathcal{U}_2)$ -(r, s)-semiopen in Y for each IF $\mathcal{T}_1$ - r -open set A in X and $f(B)$ is IF $(\mathcal{U}_2, \mathcal{U}_1)$ -(s, r)-semiopen in Y for each IF $\mathcal{T}_2$ - s -open set B in X ,
- 2) *IF pairwise (r, s) -semiclosed* if $f(A)$ is IF $(\mathcal{U}_1, \mathcal{U}_2)$ -(r, s)-semiclosed in Y for each IF $\mathcal{T}_1$ - r -closed set A in X and $f(B)$ is IF $(\mathcal{U}_2, \mathcal{U}_1)$ -(s, r)-semiclosed in Y for each IF $\mathcal{T}_2$ - s -closed set B in X .

2. Intuitionistic Fuzzy (r, s) -preopen and Preclosed Sets

Now, we define the concepts of intuitionistic fuzzy $(\mathcal{T}_i, \mathcal{T}_j)$ -(r, s)-preopen and preclosed sets, and investigate some of their properties.

Definition 2.1. Let A be an intuitionistic fuzzy set in an ISBTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $r, s \in I_0$. Then A is said to be

- 1) *IF $(\mathcal{T}_i, \mathcal{T}_j)$ -(r, s)-preopen* if $A \subseteq \mathcal{T}_i\text{-int}(\mathcal{T}_j\text{-cl}(A, s), r)$,
- 2) *IF $(\mathcal{T}_i, \mathcal{T}_j)$ -(r, s)-preclosed* if $\mathcal{T}_i\text{-cl}(\mathcal{T}_j\text{-int}(A, s), r) \subseteq A$.

Theorem 2.2. Let A be an intuitionistic fuzzy set in an ISBTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $r, s \in I_0$. Then the following statements are equivalent:

- 1) A is an IF $(\mathcal{T}_i, \mathcal{T}_j)$ -(r, s)-preopen set.
- 2) A^c is an IF $(\mathcal{T}_i, \mathcal{T}_j)$ -(r, s)-preclosed set.

Proof. 1) $\Rightarrow$ 2) Let A be an IF $(\mathcal{T}_i, \mathcal{T}_j)$ -(r, s)-preopen set. Then

$$A \subseteq \mathcal{T}_i\text{-int}(\mathcal{T}_j\text{-cl}(A, s), r).$$

Thus, we have

$$\mathcal{T}_i\text{-cl}(\mathcal{T}_j\text{-int}(A^c, s), r) = \mathcal{T}_i\text{-int}(\mathcal{T}_j\text{-cl}(A, s), r)^c \subseteq A^c.$$

Hence, A^c is an IF $(\mathcal{T}_i, \mathcal{T}_j)$ -(r, s)-preopen set.

2) $\Rightarrow$ 1) Let A^c be an IF $(\mathcal{T}_i, \mathcal{T}_j)$ -(r, s)-preclosed set. Then

$$\mathcal{T}_i\text{-cl}(\mathcal{T}_j\text{-int}(A^c, s), r) \subseteq A^c.$$

Hence, we obtain

$$A \subseteq \mathcal{T}_i\text{-cl}(\mathcal{T}_j\text{-int}(A^c, s), r)^c = \mathcal{T}_i\text{-int}(\mathcal{T}_j\text{-cl}(A, s), r).$$

Thus, A is an IF $(\mathcal{T}_i, \mathcal{T}_j)$ -(r, s)-preopen set. $\square$

Theorem 2.3. Let A be an intuitionistic fuzzy set in an ISBTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $r, s \in I_0$. Then

- 1) If A is IF $\mathcal{T}_1$ - r -open in $(X, \mathcal{T}_1)$, then A is an IF $(\mathcal{T}_1, \mathcal{T}_2)$ -(r, s)-preopen set in $(X, \mathcal{T}_1, \mathcal{T}_2)$ for all $s \in I_0$.
- 2) If A is IF $\mathcal{T}_2$ - s -open in $(X, \mathcal{T}_2)$, then A is an IF $(\mathcal{T}_2, \mathcal{T}_1)$ -(s, r)-preopen set in $(X, \mathcal{T}_1, \mathcal{T}_2)$ for all $r \in I_0$.

Proof. 1) Let A be an IF $\mathcal{T}_1$ - r -open set in $(X, \mathcal{T}_1)$. Then $A = \mathcal{T}_1\text{-int}(A, r)$. Hence, we obtain

$$A = \mathcal{T}_1\text{-int}(A, r) \subseteq \mathcal{T}_1\text{-int}(\mathcal{T}_2\text{-cl}(A, s), r).$$

Thus, A is IF $(\mathcal{T}_1, \mathcal{T}_2)$ -(r, s)-preopen in $(X, \mathcal{T}_1, \mathcal{T}_2)$.

2) Similar to 1). $\square$

The following example shows that the converses of the above theorem need not be true.

Example 2.4. Let $X = \{x, y\}$ and let A_1, A_2, A_3 , and A_4 be intuitionistic fuzzy sets in X defined as

$$\begin{aligned} A_1(x) &= (0.2, 0.7), & A_1(y) &= (0.4, 0.5); \\ A_2(x) &= (0.6, 0.3), & A_2(y) &= (0.2, 0.7); \\ A_3(x) &= (0.2, 0.7), & A_3(y) &= (0.2, 0.7); \end{aligned}$$

and

$$A_4(x) = (0.5, 0.5), \quad A_4(y) = (0.1, 0.8).$$

Define $\mathcal{T}_1 : I(X) \rightarrow I$ and $\mathcal{T}_2 : I(X) \rightarrow I$ by

$$\mathcal{T}_1(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ \frac{1}{2} & \text{if } A = A_1, \\ 0 & \text{otherwise;} \end{cases}$$

and

$$\mathcal{T}_2(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ \frac{1}{3} & \text{if } A = A_2, \\ 0 & \text{otherwise.} \end{cases}$$

Then $(\mathcal{T}_1, \mathcal{T}_2)$ is an ISBT on X . Since

$$\begin{aligned} \mathcal{T}_1\text{-int}(\mathcal{T}_2\text{-cl}(A_3, \frac{1}{3}), \frac{1}{2}) &= \mathcal{T}_1\text{-int}(A_2^c, \frac{1}{3}) \\ &= A_1 \supseteq A_3, \end{aligned}$$

A_3 is IF $(\mathcal{T}_1, \mathcal{T}_2)$ -($\frac{1}{2}, \frac{1}{3}$)-preopen in $(X, \mathcal{T}_1, \mathcal{T}_2)$. But A_3 is not an IF $\mathcal{T}_1$ - $\frac{1}{2}$ -open set in $(X, \mathcal{T}_1)$. Also we have

$$\begin{aligned} \mathcal{T}_2\text{-int}(\mathcal{T}_1\text{-cl}(A_4, \frac{1}{2}), \frac{1}{3}) &= \mathcal{T}_2\text{-int}(A_1^c, \frac{1}{3}) \\ &= A_2 \supseteq A_4, \end{aligned}$$

Thus, A_4 is IF $(\mathcal{T}_2, \mathcal{T}_1)$ -($\frac{1}{3}, \frac{1}{2}$)-preopen in $(X, \mathcal{T}_1, \mathcal{T}_2)$. But A_4 is not a IF $\mathcal{T}_2$ - $\frac{1}{3}$ -open in $(X, \mathcal{T}_2)$.

The following example shows that IF $(\mathcal{T}_i, \mathcal{T}_j)$ -(r, s)-semiopen sets and IF $(\mathcal{T}_i, \mathcal{T}_j)$ -(r, s)-preopen sets are independent notions.

Example 2.5. Let $X = \{x, y\}$ and A_1, A_2, A_3, A_4, A_5 , and A_6 be intuitionistic fuzzy sets in X defined as

$$\begin{aligned} A_1(x) &= (0.1, 0.8), & A_1(y) &= (0.6, 0.3); \\ A_2(x) &= (0.7, 0.2), & A_2(y) &= (0.2, 0.7); \\ A_3(x) &= (0, 0.9), & A_3(y) &= (0.5, 0.4); \\ A_4(x) &= (0.1, 0.8), & A_4(y) &= (0.7, 0.3); \\ A_5(x) &= (0.5, 0.5), & A_5(y) &= (0.5, 0.4); \end{aligned}$$

and

$$A_6(x) = (0.8, 0.1), \quad A_6(y) = (0.2, 0.7).$$

Define $\mathcal{T}_1 : I(X) \rightarrow I$ and $\mathcal{T}_2 : I(X) \rightarrow I$ by

$$\mathcal{T}_1(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ \frac{1}{2} & \text{if } A = A_1, \\ 0 & \text{otherwise;} \end{cases}$$

and

$$\mathcal{T}_2(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ \frac{1}{3} & \text{if } A = A_2, \\ 0 & \text{otherwise.} \end{cases}$$

Then $(\mathcal{T}_1, \mathcal{T}_2)$ is an ISBT on X . Note that

$$\begin{aligned} \mathcal{T}_1\text{-int}(\mathcal{T}_2\text{-cl}(A_3, \frac{1}{3}), \frac{1}{2}) &= \mathcal{T}_1\text{-int}(A_2^c, \frac{1}{2}) = A_1 \supseteq A_3, \\ \mathcal{T}_2\text{-cl}(\mathcal{T}_1\text{-int}(A_3, \frac{1}{2}), \frac{1}{3}) &= \mathcal{T}_2\text{-cl}(\underline{0}, \frac{1}{3}) = \underline{0} \not\supseteq A_3, \\ \mathcal{T}_2\text{-cl}(\mathcal{T}_1\text{-int}(A_4, \frac{1}{2}), \frac{1}{3}) &= \mathcal{T}_2\text{-cl}(A_1, \frac{1}{3}) = A_2^c \supseteq A_4, \\ \mathcal{T}_1\text{-int}(\mathcal{T}_2\text{-cl}(A_4, \frac{1}{3}), \frac{1}{2}) &= \mathcal{T}_1\text{-int}(A_2^c, \frac{1}{2}) = A_1 \not\supseteq A_4, \\ \mathcal{T}_2\text{-int}(\mathcal{T}_1\text{-cl}(A_5, \frac{1}{2}), \frac{1}{3}) &= \mathcal{T}_2\text{-int}(\underline{1}, \frac{1}{3}) = \underline{1} \supseteq A_5, \\ \mathcal{T}_1\text{-cl}(\mathcal{T}_2\text{-int}(A_5, \frac{1}{3}), \frac{1}{2}) &= \mathcal{T}_1\text{-cl}(\underline{0}, \frac{1}{2}) = \underline{0} \not\supseteq A_5, \\ \mathcal{T}_1\text{-cl}(\mathcal{T}_2\text{-int}(A_6, \frac{1}{3}), \frac{1}{2}) &= \mathcal{T}_1\text{-cl}(A_2, \frac{1}{2}) = \underline{1} \supseteq A_6, \end{aligned}$$

and

$$\mathcal{T}_2\text{-int}(\mathcal{T}_1\text{-cl}(A_6, \frac{1}{2}), \frac{1}{3}) = \mathcal{T}_2\text{-int}(A_1^c, \frac{1}{3}) = A_2 \not\supseteq A_6.$$

Hence, A_3 is an IF $(\mathcal{T}_1, \mathcal{T}_2)$ - $(\frac{1}{2}, \frac{1}{3})$ -preopen set which is not IF $(\mathcal{T}_1, \mathcal{T}_2)$ - $(\frac{1}{2}, \frac{1}{3})$ -semiopen. Also, A_4 is an IF $(\mathcal{T}_1, \mathcal{T}_2)$ - $(\frac{1}{2}, \frac{1}{3})$ -semiopen set which is not IF $(\mathcal{T}_1, \mathcal{T}_2)$ - $(\frac{1}{2}, \frac{1}{3})$ -preopen. The set A_5 is an IF $(\mathcal{T}_2, \mathcal{T}_1)$ - $(\frac{1}{3}, \frac{1}{2})$ -preopen set which is not IF $(\mathcal{T}_2, \mathcal{T}_1)$ - $(\frac{1}{3}, \frac{1}{2})$ -semiopen. Also, A_6 is an IF $(\mathcal{T}_2, \mathcal{T}_1)$ - $(\frac{1}{3}, \frac{1}{2})$ -semiopen set which is not IF $(\mathcal{T}_2, \mathcal{T}_1)$ - $(\frac{1}{3}, \frac{1}{2})$ -preopen.

Theorem 2.6. Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be an ISBTS and $r, s \in I_0$. Then the following statements are true:

- 1) If $\{A_k\}$ is a family of IF $(\mathcal{T}_i, \mathcal{T}_j)$ - (r, s) -preopen sets in X , then $\bigcup A_k$ is IF $(\mathcal{T}_i, \mathcal{T}_j)$ - (r, s) -preopen.
- 2) If $\{A_k\}$ is a family of IF $(\mathcal{T}_i, \mathcal{T}_j)$ - (r, s) -preclosed sets in X , then $\bigcap A_k$ is IF $(\mathcal{T}_i, \mathcal{T}_j)$ - (r, s) -preclosed.

Proof. (1) Let $\{A_k\}$ be a collection of IF $(\mathcal{T}_i, \mathcal{T}_j)$ - (r, s) -preopen sets in X . Then for each k ,

$$A_k \subseteq \mathcal{T}_i\text{-int}(\mathcal{T}_j\text{-cl}(A_k, s), r).$$

Thus, we have

$$\begin{aligned} \bigcup A_k &\subseteq \bigcup \mathcal{T}_i\text{-int}(\mathcal{T}_j\text{-cl}(A_k, s), r) \\ &\subseteq \mathcal{T}_i\text{-int}(\mathcal{T}_j\text{-cl}(\bigcup A_k, s), r). \end{aligned}$$

Thus, $\bigcup A_k$ is IF $(\mathcal{T}_i, \mathcal{T}_j)$ - (r, s) -preopen.

2) It follows from (1) using Theorem 2.2. $\square$

Definition 2.7. Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be an ISBTS and $r, s \in I_0$. For each $A \in I(X)$, the IF $(\mathcal{T}_i, \mathcal{T}_j)$ - (r, s) -preinterior is defined by

$$\begin{aligned} (\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(A, r, s) \\ = \bigcup \{B \in I(X) \mid B \subseteq A, B \text{ is IF } (\mathcal{T}_i, \mathcal{T}_j)\text{-}(r, s)\text{-preopen}\}, \end{aligned}$$

and the IF $(\mathcal{T}_i, \mathcal{T}_j)$ - (r, s) -preclosure is defined by

$$\begin{aligned} (\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(A, r, s) \\ = \bigcap \{B \in I(X) \mid A \subseteq B, B \text{ is IF } (\mathcal{T}_i, \mathcal{T}_j)\text{-}(r, s)\text{-preclosed}\}. \end{aligned}$$

Obviously, $(\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(A, r, s)$ is the smallest IF $(\mathcal{T}_i, \mathcal{T}_j)$ - (r, s) -preclosed set which contains A , and $(\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(A, r, s)$ is the greatest IF $(\mathcal{T}_i, \mathcal{T}_j)$ - (r, s) -preopen set which is contained in A . Clearly, $(\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(A, r, s) = A$ for any IF $(\mathcal{T}_i, \mathcal{T}_j)$ - (r, s) -preclosed set A , and $(\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(A, r, s) = A$ for any IF $(\mathcal{T}_i, \mathcal{T}_j)$ - (r, s) -preopen set A .

Moreover, we have

$$\begin{aligned} \mathcal{T}_i\text{-int}(A, r) &\subseteq (\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(A, r, s) \\ &\subseteq A \\ &\subseteq (\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(A, r, s) \\ &\subseteq \mathcal{T}_i\text{-cl}(A, r). \end{aligned}$$

Also, we have the following results:

- 1) $(\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(\underline{0}, r, s) = \underline{0}$, $(\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(\underline{1}, r, s) = \underline{1}$.
- 2) $(\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(A, r, s) \supseteq A$.
- 3) $(\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(A, r, s) \cup (\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(B, r, s) \subseteq (\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(A \cup B, r, s)$.
- 4) $(\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}((\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(A, r, s), r, s) = (\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(A, r, s)$.
- 5) $(\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(\underline{0}, r, s) = \underline{0}$, $(\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(\underline{1}, r, s) = \underline{1}$.
- 6) $(\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(A, r, s) \subseteq A$.
- 7) $(\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(A, r, s) \cap (\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(B, r, s) \supseteq (\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(A \cap B, r, s)$.
- 8) $(\mathcal{T}_i, \mathcal{T}_j)\text{-pint}((\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(A, r, s), r, s) = (\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(A, r, s)$.

Theorem 2.8. Let A be an intuitionistic fuzzy set in an ISBTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $r, s \in I_0$. Then we have

- 1) $(\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(A, r, s)^c = (\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(A^c, r, s)$.
- 2) $(\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(A, r, s)^c = (\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(A^c, r, s)$.

Proof. From $(\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(A, r, s) \subseteq A$, we have that $A^c \subseteq (\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(A, r, s)^c$. Since $(\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(A, r, s)$ is IF $(\mathcal{T}_i, \mathcal{T}_j)\text{-(}r, s\text{)-preopen}$ in X , the set $(\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(A, r, s)^c$ is IF $(\mathcal{T}_i, \mathcal{T}_j)\text{-(}r, s\text{)-preclosed}$. Hence,

$$\begin{aligned} & (\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(A^c, r, s) \\ & \subseteq (\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}((\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(A, r, s)^c, r, s) \\ & = (\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(A, r, s)^c. \end{aligned}$$

From the fact that $A^c \subseteq (\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(A^c, r, s)$, we have $(\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(A^c, r, s)^c \subseteq A$. Since $(\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(A^c, r, s)$ is IF $(\mathcal{T}_i, \mathcal{T}_j)\text{-(}r, s\text{)-preclosed}$, $(\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(A^c, r, s)^c$ is IF $(\mathcal{T}_i, \mathcal{T}_j)\text{-(}r, s\text{)-preopen}$. Hence, we obtain that

$$\begin{aligned} & (\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(A^c, r, s)^c \\ & = (\mathcal{T}_i, \mathcal{T}_j)\text{-pint}((\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(A^c, r, s)^c, r, s) \\ & \subseteq (\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(A, r, s). \end{aligned}$$

Thus,

$$(\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(A, r, s)^c \subseteq (\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(A^c, r, s).$$

Therefore,

$$(\mathcal{T}_i, \mathcal{T}_j)\text{-pint}(A, r, s)^c = (\mathcal{T}_i, \mathcal{T}_j)\text{-pcl}(A^c, r, s).$$

2) Similar to 1). $\square$

3. Intuitionistic Fuzzy Pairwise (r, s) -Precontinuous Mappings

We define the notion of intuitionistic fuzzy pairwise (r, s) -precontinuous mappings in intuitionistic smooth bitopological spaces, and investigate some of their characteristic properties.

Definition 3.1. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from an ISBTS X to an ISBTS Y and $r, s \in I_0$. Then f is said to be *IF pairwise (r, s) -precontinuous* if $f^{-1}(A)$ is an IF $(\mathcal{T}_1, \mathcal{T}_2)\text{-(}r, s\text{)-preopen}$ set in X for each IF $\mathcal{U}_1$ - r -open set A in Y and $f^{-1}(B)$ is an IF $(\mathcal{T}_2, \mathcal{T}_1)\text{-(}s, r\text{)-preopen}$ set in X for each IF $\mathcal{U}_2$ - s -open set B in Y .

Remark 3.2. It is obvious that every IF pairwise (r, s) -continuous mapping is IF pairwise (r, s) -precontinuous. However, the following example shows that the converse need not be true.

Example 3.3. Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be an ISBTS as described in Example 2.4.

Define $\mathcal{U}_1 : I(X) \rightarrow I$ and $\mathcal{U}_2 : I(X) \rightarrow I$ by

$$\mathcal{U}_1(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ \frac{1}{2} & \text{if } A = A_3, \\ 0 & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}_2(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ 0 & \text{otherwise.} \end{cases}$$

Then $(\mathcal{U}_1, \mathcal{U}_2)$ is an ISBT on X . Consider a mapping $f : (X, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (X, \mathcal{U}_1, \mathcal{U}_2)$ defined by $f(x) = x$ and $f(y) = y$. Then f is IF pairwise $(\frac{1}{2}, \frac{1}{3})$ -precontinuous. But f is not an IF pairwise $(\frac{1}{2}, \frac{1}{3})$ -continuous mapping.

The following examples show that intuitionistic fuzzy pairwise (r, s) -semicontinuous mappings and IF pairwise (r, s) -precontinuous mappings are independent notions

Example 3.4. Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be an ISBTS as described in Example 2.4.

Define $\mathcal{U}_1 : I(X) \rightarrow I$ and $\mathcal{U}_2 : I(X) \rightarrow I$ by

$$\mathcal{U}_1(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ 0 & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}_2(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ \frac{1}{3} & \text{if } A = A_5, \\ 0 & \text{otherwise.} \end{cases}$$

Then $(\mathcal{U}_1, \mathcal{U}_2)$ is an ISBT on X . Consider a mapping $g : (X, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (X, \mathcal{U}_1, \mathcal{U}_2)$ defined by $g(x) = x$ and $g(y) = y$. Then g is an IF pairwise $(\frac{1}{2}, \frac{1}{3})$ -precontinuous mapping. But g is not IF pairwise $(\frac{1}{2}, \frac{1}{3})$ -semicontinuous.

Example 3.5. Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be an ISBTS as described in Example 2.4.

Define $\mathcal{U}_1 : I(X) \rightarrow I$ and $\mathcal{U}_2 : I(X) \rightarrow I$ by

$$\mathcal{U}_1(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ 0 & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}_2(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ \frac{1}{3} & \text{if } A = A_6, \\ 0 & \text{otherwise.} \end{cases}$$

Then $(\mathcal{U}_1, \mathcal{U}_2)$ is an ISBT on X . Consider a mapping $h : (X, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (X, \mathcal{U}_1, \mathcal{U}_2)$ defined by $h(x) = x$ and $h(y) = y$. Then h is an IF pairwise $(\frac{1}{2}, \frac{1}{3})$ -semicontinuous mapping. But h is not IF pairwise $(\frac{1}{2}, \frac{1}{3})$ -precontinuous.

We can characterize the IF pairwise precontinuity by the property of inverse image of a closed set, and also by the relations of interior and closure.

Theorem 3.6. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from an ISBTS X to an ISBTS Y and $r, s \in I_0$. Then the following statements are equivalent:

- 1) f is IF pairwise (r, s) -precontinuous.
- 2) $f^{-1}(A)$ is an IF $(\mathcal{T}_1, \mathcal{T}_2)$ -(r, s)-preclosed set in X for each IF $\mathcal{U}_1$ - r -closed set A in Y and $f^{-1}(B)$ is an IF $(\mathcal{T}_2, \mathcal{T}_1)$ -(s, r)-preclosed set in X for each IF $\mathcal{U}_2$ - s -closed set B in Y .
- 3) For each intuitionistic fuzzy set B in Y ,

$$\mathcal{T}_1\text{-cl}(\mathcal{T}_2\text{-int}(f^{-1}(B), s), r) \subseteq f^{-1}(\mathcal{U}_1\text{-cl}(B, r)),$$

and

$$\mathcal{T}_2\text{-cl}(\mathcal{T}_1\text{-int}(f^{-1}(B), r), s) \subseteq f^{-1}(\mathcal{U}_2\text{-cl}(B, s)).$$

- (4) For each intuitionistic fuzzy set A in X ,

$$f(\mathcal{T}_1\text{-cl}(\mathcal{T}_2\text{-int}(A, s), r)) \subseteq \mathcal{U}_1\text{-cl}(f(A), r),$$

and

$$f(\mathcal{T}_2\text{-cl}(\mathcal{T}_1\text{-int}(A, r), s)) \subseteq \mathcal{U}_2\text{-cl}(f(A), s).$$

Proof. 1) $\Leftrightarrow$ 2) Trivial.

2) $\Rightarrow$ 3) Let B be an intuitionistic fuzzy set in Y . Then $\mathcal{U}_1\text{-cl}(B, r)$ is IF $\mathcal{U}_1$ - r -closed and $\mathcal{U}_2\text{-cl}(B, s)$ is IF $\mathcal{U}_2$ - s -closed in Y . Thus, by 2), $f^{-1}(\mathcal{U}_1\text{-cl}(B, r))$ is an IF $(\mathcal{T}_1, \mathcal{T}_2)$ -(r, s)-preclosed set and $f^{-1}(\mathcal{U}_2\text{-cl}(B, s))$ is an IF $(\mathcal{T}_2, \mathcal{T}_1)$ -(s, r)-preclosed set in X . Hence, we have

$$\mathcal{T}_1\text{-cl}(\mathcal{T}_2\text{-int}(f^{-1}(B), s), r)$$

$$\begin{aligned} &\subseteq \mathcal{T}_1\text{-cl}(\mathcal{T}_2\text{-int}(f^{-1}(\mathcal{U}_1\text{-cl}(B, r)), s), r) \\ &\subseteq f^{-1}(\mathcal{U}_1\text{-cl}(B, r)) \end{aligned}$$

and

$$\begin{aligned} &\mathcal{T}_2\text{-cl}(\mathcal{T}_1\text{-int}(f^{-1}(B), r), s) \\ &\subseteq \mathcal{T}_2\text{-cl}(\mathcal{T}_1\text{-int}(f^{-1}(\mathcal{U}_2\text{-cl}(B, s)), r), s) \\ &\subseteq f^{-1}(\mathcal{U}_2\text{-cl}(B, s)). \end{aligned}$$

3) $\Rightarrow$ 4) Let A be an intuitionistic fuzzy set in X . Then by 3), we obtain

$$\begin{aligned} \mathcal{T}_1\text{-cl}(\mathcal{T}_2\text{-int}(A, s), r) &\subseteq \mathcal{T}_1\text{-cl}(\mathcal{T}_2\text{-int}(f^{-1}(f(A)), s), r) \\ &\subseteq f^{-1}(\mathcal{U}_1\text{-cl}(f(A), r)), \end{aligned}$$

and

$$\begin{aligned} \mathcal{T}_2\text{-cl}(\mathcal{T}_1\text{-int}(A, r), s) &\subseteq \mathcal{T}_2\text{-cl}(\mathcal{T}_1\text{-int}(f^{-1}(f(A)), r), s) \\ &\subseteq f^{-1}(\mathcal{U}_2\text{-cl}(f(A), s)). \end{aligned}$$

Hence

$$f(\mathcal{T}_1\text{-cl}(\mathcal{T}_2\text{-int}(A, s), r)) \subseteq \mathcal{U}_1\text{-cl}(f(A), r),$$

and

$$f(\mathcal{T}_2\text{-cl}(\mathcal{T}_1\text{-int}(A, r), s)) \subseteq \mathcal{U}_2\text{-cl}(f(A), s).$$

4) $\Rightarrow$ 2) Let A be any IF $\mathcal{U}_1$ - r -closed set and B any IF $\mathcal{U}_2$ - s -closed set in Y . By (4), we have

$$\begin{aligned} f(\mathcal{T}_1\text{-cl}(\mathcal{T}_2\text{-int}(f^{-1}(A), s), r)) &\subseteq \mathcal{U}_1\text{-cl}(f(f^{-1}(A)), r) \\ &\subseteq \mathcal{U}_1\text{-cl}(A, r) = A, \end{aligned}$$

and

$$\begin{aligned} f(\mathcal{T}_2\text{-cl}(\mathcal{T}_1\text{-int}(f^{-1}(B), r), s)) &\subseteq \mathcal{U}_2\text{-cl}(f(f^{-1}(B)), s) \\ &\subseteq \mathcal{U}_2\text{-cl}(B, s) = B. \end{aligned}$$

Hence

$$\mathcal{T}_1\text{-cl}(\mathcal{T}_2\text{-int}(f^{-1}(A), s), r) \subseteq f^{-1}(A),$$

and

$$\mathcal{T}_2\text{-cl}(\mathcal{T}_1\text{-int}(f^{-1}(B), r), s) \subseteq f^{-1}(B).$$

Therefore, $f^{-1}(A)$ is an IF $(\mathcal{T}_1, \mathcal{T}_2)$ -(r, s)-preclosed set and $f^{-1}(B)$ is an IF $(\mathcal{T}_2, \mathcal{T}_1)$ -(s, r)-preclosed set in X . $\square$

We can characterize the IF pairwise precontinuity by the relations of preinterior or preclosure.

Theorem 3.7. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from an ISBTS X to an ISBTS Y and $r, s \in I_0$. Then the following statements are equivalent:

- 1) f is IF pairwise (r, s) -precontinuous.
- 2) For each intuitionistic fuzzy set A in X ,

$$f((\mathcal{T}_1, \mathcal{T}_2)\text{-pcl}(A, r, s)) \subseteq \mathcal{U}_1\text{-cl}(f(A), r),$$

and

$$f((\mathcal{T}_2, \mathcal{T}_1)\text{-pcl}(A, s, r)) \subseteq \mathcal{U}_2\text{-cl}(f(A), r).$$

- (3) For each intuitionistic fuzzy set B in Y ,

$$(\mathcal{T}_1, \mathcal{T}_2)\text{-pcl}(f^{-1}(B), r, s) \subseteq f^{-1}(\mathcal{U}_1\text{-cl}(B, r)),$$

and

$$(\mathcal{T}_2, \mathcal{T}_1)\text{-pcl}(f^{-1}(B), s, r) \subseteq f^{-1}(\mathcal{U}_2\text{-cl}(B, s)).$$

- (4) For each intuitionistic fuzzy set B in Y ,

$$f^{-1}(\mathcal{U}_1\text{-int}(B, r)) \subseteq (\mathcal{T}_1, \mathcal{T}_2)\text{-pint}(f^{-1}(B), r, s)$$

and

$$f^{-1}(\mathcal{U}_2\text{-int}(B, s)) \subseteq (\mathcal{T}_2, \mathcal{T}_1)\text{-pint}(f^{-1}(B), s, r).$$

Proof. 1) $\Rightarrow$ 2) Let A be an intuitionistic fuzzy set in X . Then $\mathcal{U}_1\text{-cl}(f(A), r)$ is IF $\mathcal{U}_1$ - r -closed and $\mathcal{U}_2\text{-cl}(f(A), s)$ is IF $\mathcal{U}_2$ - s -closed in Y . Since f is IF pairwise (r, s) -precontinuous, $f^{-1}(\mathcal{U}_1\text{-cl}(f(A), r))$ is an IF $(\mathcal{T}_1, \mathcal{T}_2)$ - (r, s) -preclosed set and $f^{-1}(\mathcal{U}_2\text{-cl}(f(A), s))$ is an IF $(\mathcal{T}_2, \mathcal{T}_1)$ - (s, r) -preclosed set in X . Thus

$$\begin{aligned} & (\mathcal{T}_1, \mathcal{T}_2)\text{-pcl}(A, r, s) \\ & \subseteq (\mathcal{T}_1, \mathcal{T}_2)\text{-pcl}(f^{-1}(\mathcal{U}_1\text{-cl}(f(A), r)), r, s) \\ & = f^{-1}(\mathcal{U}_1\text{-cl}(f(A), r)), \end{aligned}$$

and

$$\begin{aligned} & (\mathcal{T}_2, \mathcal{T}_1)\text{-pcl}(A, s, r) \\ & \subseteq (\mathcal{T}_2, \mathcal{T}_1)\text{-pcl}(f^{-1}(\mathcal{U}_2\text{-cl}(f(A), s)), s, r) \\ & = f^{-1}(\mathcal{U}_2\text{-cl}(f(A), s)). \end{aligned}$$

Therefore

$$f((\mathcal{T}_1, \mathcal{T}_2)\text{-pcl}(A, r, s)) \subseteq \mathcal{U}_1\text{-cl}(f(A), r),$$

and

$$f((\mathcal{T}_2, \mathcal{T}_1)\text{-pcl}(A, s, r)) \subseteq \mathcal{U}_2\text{-cl}(f(A), s).$$

(2) $\Rightarrow$ (3) Let B be an intuitionistic fuzzy set in Y . Then by (2), we have

$$\begin{aligned} f((\mathcal{T}_1, \mathcal{T}_2)\text{-pcl}(f^{-1}(B), r, s)) & \subseteq \mathcal{U}_1\text{-cl}(f(f^{-1}(B)), r) \\ & \subseteq \mathcal{U}_1\text{-cl}(B, r), \end{aligned}$$

and

$$\begin{aligned} f((\mathcal{T}_2, \mathcal{T}_1)\text{-pcl}(f^{-1}(B), s, r)) & \subseteq \mathcal{U}_2\text{-cl}(f(f^{-1}(B)), s) \\ & \subseteq \mathcal{U}_2\text{-cl}(B, s). \end{aligned}$$

Hence

$$(\mathcal{T}_1, \mathcal{T}_2)\text{-pcl}(f^{-1}(B), r, s) \subseteq f^{-1}(\mathcal{U}_1\text{-cl}(B, r))$$

and

$$(\mathcal{T}_2, \mathcal{T}_1)\text{-pcl}(f^{-1}(B), s, r) \subseteq f^{-1}(\mathcal{U}_2\text{-cl}(B, s)).$$

(3) $\Rightarrow$ (4) Let B be an intuitionistic fuzzy set in Y . Then by (3), we have

$$(\mathcal{T}_1, \mathcal{T}_2)\text{-pcl}(f^{-1}(B^c), r, s) \subseteq f^{-1}(\mathcal{U}_1\text{-cl}(B^c, r)),$$

and

$$(\mathcal{T}_2, \mathcal{T}_1)\text{-pcl}(f^{-1}(B^c), s, r) \subseteq f^{-1}(\mathcal{U}_2\text{-cl}(B^c, s)).$$

Hence

$$\begin{aligned} f^{-1}(\mathcal{U}_1\text{-int}(B, r)) & = (f^{-1}(\mathcal{U}_1\text{-cl}(B^c, r)))^c \\ & \subseteq (\mathcal{T}_1, \mathcal{T}_2)\text{-pcl}(f^{-1}(B^c), r, s)^c \\ & = (\mathcal{T}_1, \mathcal{T}_2)\text{-pint}(f^{-1}(B), r, s), \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\mathcal{U}_2\text{-int}(B, s)) & = (f^{-1}(\mathcal{U}_2\text{-cl}(B^c, s)))^c \\ & \subseteq (\mathcal{T}_2, \mathcal{T}_1)\text{-pcl}(f^{-1}(B^c), s, r)^c \\ & = (\mathcal{T}_2, \mathcal{T}_1)\text{-pint}(f^{-1}(B), s, r). \end{aligned}$$

4) $\Rightarrow$ 1) Let A be any IF $\mathcal{U}_1$ - r -open set and B any IF $\mathcal{U}_2$ - s -

open set in Y . Then $\mathcal{U}_1\text{-int}(A, r) = A$ and $\mathcal{U}_2\text{-int}(B, s) = B$.

Hence

$$\begin{aligned} f^{-1}(A) &= f^{-1}(\mathcal{U}_1\text{-int}(A, r)) \\ &\subseteq (\mathcal{T}_1, \mathcal{T}_2)\text{-pint}(f^{-1}(A), r, s) \\ &\subseteq f^{-1}(A), \end{aligned}$$

and

$$\begin{aligned} f^{-1}(B) &= f^{-1}(\mathcal{U}_2\text{-int}(B, s)) \\ &\subseteq (\mathcal{T}_2, \mathcal{T}_1)\text{-pint}(f^{-1}(B), s, r) \\ &\subseteq f^{-1}(B). \end{aligned}$$

Hence

$$f^{-1}(A) = (\mathcal{T}_1, \mathcal{T}_2)\text{-pint}(f^{-1}(A), r, s),$$

and

$$f^{-1}(B) = (\mathcal{T}_2, \mathcal{T}_1)\text{-pint}(f^{-1}(B), s, r).$$

Thus, $f^{-1}(A)$ is an IF $(\mathcal{T}_1, \mathcal{T}_2)$ -(r, s)-preopen set and $f^{-1}(B)$ is an IF $(\mathcal{T}_2, \mathcal{T}_1)$ -(s, r)-preopen set in X . Therefore, f is IF pairwise (r, s) -precontinuous. $\square$

Theorem 3.8. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a bijective mapping from an ISBTS X to an ISBTS Y and $r, s \in I_0$. Then f is IF pairwise (r, s) -precontinuous if and only if

$$\mathcal{U}_1\text{-int}(f(A), r) \subseteq f((\mathcal{T}_1, \mathcal{T}_2)\text{-pint}(A, r, s)),$$

and

$$\mathcal{U}_2\text{-int}(f(A), s) \subseteq f((\mathcal{T}_2, \mathcal{T}_1)\text{-pint}(A, s, r))$$

for each intuitionistic fuzzy set A in X .

Proof. Let A be an intuitionistic fuzzy set in X . Since f is one-to-one, by Theorem 3.7, we obtain

$$\begin{aligned} f^{-1}(\mathcal{U}_1\text{-int}(f(A), r)) &\subseteq (\mathcal{T}_1, \mathcal{T}_2)\text{-pint}(f^{-1}(f(A)), r, s) \\ &= (\mathcal{T}_1, \mathcal{T}_2)\text{-pint}(A, r, s), \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\mathcal{U}_2\text{-int}(f(A), s)) &\subseteq (\mathcal{T}_2, \mathcal{T}_1)\text{-pint}(f^{-1}(f(A)), s, r) \\ &= (\mathcal{T}_2, \mathcal{T}_1)\text{-pint}(A, s, r). \end{aligned}$$

Since f is onto, we have

$$\mathcal{U}_1\text{-int}(f(A), r) = f(f^{-1}(\mathcal{U}_1\text{-int}(f(A), r)))$$

$$\subseteq f((\mathcal{T}_1, \mathcal{T}_2)\text{-pint}(A, r, s)),$$

and

$$\begin{aligned} \mathcal{U}_2\text{-int}(f(A), s) &= f(f^{-1}(\mathcal{U}_2\text{-int}(f(A), s))) \\ &\subseteq f((\mathcal{T}_2, \mathcal{T}_1)\text{-pint}(A, s, r)). \end{aligned}$$

Conversely, let B be an intuitionistic fuzzy set in Y . Since f is onto, we have

$$\begin{aligned} \mathcal{U}_1\text{-int}(B, r) &= \mathcal{U}_1\text{-int}(f(f^{-1}(B)), r) \\ &\subseteq f((\mathcal{T}_1, \mathcal{T}_2)\text{-pint}(f^{-1}(B), r, s)), \end{aligned}$$

and

$$\begin{aligned} \mathcal{U}_2\text{-int}(B, s) &= \mathcal{U}_2\text{-int}(f(f^{-1}(B)), s) \\ &\subseteq f((\mathcal{T}_2, \mathcal{T}_1)\text{-pint}(f^{-1}(B), s, r)). \end{aligned}$$

Since f is one-to-one, we obtain

$$\begin{aligned} f^{-1}(\mathcal{U}_1\text{-int}(B, r)) &\subseteq f^{-1}(f((\mathcal{T}_1, \mathcal{T}_2)\text{-pint}(f^{-1}(B), r, s))) \\ &= (\mathcal{T}_1, \mathcal{T}_2)\text{-pint}(f^{-1}(B), r, s), \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\mathcal{U}_2\text{-int}(B, s)) &\subseteq f^{-1}(f((\mathcal{T}_2, \mathcal{T}_1)\text{-pint}(f^{-1}(B), s, r))) \\ &= (\mathcal{T}_2, \mathcal{T}_1)\text{-pint}(f^{-1}(B), s, r). \end{aligned}$$

Therefore, by Theorem 3.7, f is an IF pairwise (r, s) -precontinuous mapping. $\square$

4. Conclusions

In this paper, we introduced a new notion of intuitionistic fuzzy pairwise precontinuous mappings in intuitionistic smooth bitopological spaces. By investigating the relations of closedness, interior, closure, preinterior, and preclosure, we obtained six equivalent conditions for intuitionistic fuzzy pairwise precontinuous mappings in Theorem 3.6 and Theorem 3.7. Furthermore, we obtained another equivalent condition for a bijective mapping in Theorem 3.8.

Acknowledgments

This work was conducted during the research year of Chungbuk National University in 2018.

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