An Intelligent Similarity Model between Generalized Trapezoidal Fuzzy Numbers in Large Scale

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Abstract

The rapid expansion of data published on the web has given rise to the similarity problem on a large scale, a very important subject for scientific research in the field of computer science. Several methods have been developed for this. In this paper, we propose the first mathematical model to find the similarity value between generalized trapezoidal fuzzy numbers (GTFNs). This model employs fuzzy inference systems to find the value of an effective weighting, the weights to be associated to different kinds of methods that can handle an important scale of the data. This model will allow us to develop intelligent systems. A comparative study based on 21 sets of GTFNs has been carried out to demonstrate the difference between our approach and existing methods. This study shows that our model is more reasonable than existing methods.

Keywords: Cosine coefficient, Jaccard index, GTFNs, FIS, Similarity, Large scale

1. Introduction

The concept of fuzzy logic, proposed in 1965 by Zadeh [1], has been used to manage a kind of probability. To employ this concept, the generalized trapezoidal fuzzy numbers (GTFNs) are most the most popular in practice. In the literature, several similarity methods between GTFNs have been introduced (e.g., [2–10]). But, these existing methods of similarity measures have many weaknesses. In many situations, such methods cannot appropriately find the similarity between two GTFNs. In the present study, a novel mathematical model for a fuzzy-number similarity method between GTFNs has been created, based on the weights associated to each similarity measure. This model uses the cosine coefficient and the Jaccard Index. Additionally, we describe and provide three characteristics of the proposed model. A comparative study has been carried out, based on 21 sets of GTFNs, to show that the model can surmount the limitations of the existing measures.

The remainder of this article is structured as follows. Section 2 presents a summary of the fundamental notions of existing methods. Section 3 introduces the novel approach, presenting the similarity methods employed and finding the weights associated to each similarity method. Many properties are suggested and proven. Section 4 compares it with the existing similarity methods. We give our conclusions in Section 5.

2. Related Work

The notion of a GFN $\tilde{T}$ is presented by Chen [11, 12] as follows: $\tilde{T} = \{t_1, t_2, t_3, t_4, w_{\tilde{T}}\}$, where

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the t_k $k = 1, 2, 3, 4$ denote real numbers, and we take into account a weight $w_{\tilde{T}}$ with $0 < w_{\tilde{T}} < 1$. When $w_{\tilde{T}} = 1$, $\tilde{T}$ is a normal trapezoidal fuzzy number (NTFN). When $t_2 = t_3$, $\tilde{T}$ is called a GTFN. When $t_1 = t_2$ and $t_3 = t_4$, $\tilde{T}$ is called a crisp interval. When $w_{\tilde{T}} = 1$ and $t_1 = t_k$ (where $k = 1, 2, 3, 4$), $\tilde{T}$ is a real number.

Several Similarity methods between GTFNs $\tilde{T}$ and $\tilde{H} = \{h_1, h_2, h_3, h_4, w_{\tilde{H}}\}$ have been introduced:

Chen [2] proposed a new method to solve the similarity problems of the GFNs. This method is based on the geometric distance. The author used this method to clear the problems of fuzzy risk analysis and subjective mental workload assessment.

$$S_C(\tilde{T}, \tilde{H}) = 1 - \left(\sum_{k=1}^4 \frac{t_k - h_k}{4} \right). \quad (1)$$

Hsieh and Chen [3] introduced a new approach to find a solution to the similarity problems of the GFNs. This approach is based on the average integration representation distance.

$$S_{HC}(\tilde{T}, \tilde{H}) = \frac{1}{1 + d(\tilde{T}, \tilde{H})}, \quad (2)$$

where $d(\tilde{T}, \tilde{H}) = |P(\tilde{T}) - P(\tilde{H})|$, $P(\tilde{T}) = \frac{t_1 + 2t_2 + 2t_3 + t_4}{6}$, $P(\tilde{H}) = \frac{h_1 + 2h_2 + 2h_3 + h_4}{6}$.

Lee [4] presented a new measure to answer the similarity problems of the GFNs. This measure is based on the the L_P norm. The author used this measure to solve the problems of aggregating individual opinions into group consensus under group decision.

$$S_L(\tilde{T}, \tilde{H}) = 1 - \frac{\|\tilde{T} - \tilde{H}\|_{L_P}}{\|U\|} \times 4^{-\frac{1}{P}}, \quad (3)$$

where $\|\tilde{T} - \tilde{H}\|_{L_P} = (\sum_{k=1}^4 (|t_k - h_k|)^P)^{\frac{1}{P}}$, taking into account a positive integer P . $\|U\| = \max U - \min U$ where U indicates the universe of discourse. Chen and Chen [5] defined a new proposed method to clear the similarity problems of GFNs. It is based on the simple center of gravity method (SCGM). The authors used this proposed to solve the problems of fuzzy risk analysis.

$$S_{CC}(\tilde{T}, \tilde{H}) = 1 - \left(\sum_{k=1}^4 \frac{t_k - h_k}{4} \right) \times (1 - |X_{\tilde{T}}^* - X_{\tilde{H}}^*|)^{B(S_{\tilde{T}}, S_{\tilde{H}})} \times \frac{\min(y_{\tilde{T}}^*, y_{\tilde{H}}^*)}{\max(y_{\tilde{T}}^*, y_{\tilde{H}}^*)}, \quad (4)$$

where

$$y_{\tilde{T}}^* = \begin{cases} \frac{w_{\tilde{T}}}{6} \times (2 + \frac{t_3 - t_2}{t_4 - t_1}) & \text{if } t_4 \neq t_1, \\ \frac{w_{\tilde{T}}}{2} & \text{if } t_4 = t_1, \end{cases}$$

$$y_{\tilde{H}}^* = \begin{cases} \frac{w_{\tilde{H}}}{6} \times (2 + \frac{h_3 - h_2}{h_4 - h_1}) & \text{if } h_4 \neq h_1, \\ \frac{w_{\tilde{H}}}{2} & \text{if } h_4 = h_1, \end{cases}$$

$$x_{\tilde{T}}^* = \begin{cases} \frac{(y_{\tilde{T}}^* \times (t_3 + t_2)) + (t_4 + t_1) \times (w_{\tilde{T}} - y_{\tilde{T}}^*)}{2w_{\tilde{T}}} & \text{if } w_{\tilde{T}} \neq 0, \\ \frac{t_4 + t_1}{2} & \text{if } w_{\tilde{T}} = 0, \end{cases}$$

$$x_{\tilde{H}}^* = \begin{cases} \frac{(y_{\tilde{H}}^* \times (h_3 + h_2)) + (h_4 + h_1) \times (w_{\tilde{H}} - y_{\tilde{H}}^*)}{2w_{\tilde{H}}} & \text{if } w_{\tilde{H}} \neq 0, \\ \frac{h_4 + h_1}{2} & \text{if } w_{\tilde{H}} = 0, \end{cases}$$

$$B(S_{\tilde{T}}, S_{\tilde{H}}) = \begin{cases} 1 & \text{if } S_{\tilde{T}} + S_{\tilde{H}} > 0, \\ 0 & \text{if } S_{\tilde{T}} + S_{\tilde{H}} = 0, \end{cases}$$

where $S_{\tilde{T}} = t_4 - t_1$, $S_{\tilde{H}} = h_4 - h_1$. Wei and Chen [7] proposed a new approach in order to solve the similarity problems of GFNs. It is based on the concepts of geometric distance, the perimeter and the height of GFNs. The authors used this approach to answer the problems of fuzzy risk analysis.

$$S_{WC}(\tilde{T}, \tilde{H}) = (1 - (\sum_{k=1}^4 \frac{t_k - h_k}{4})) \times (\frac{\min(P(\tilde{T}), P(\tilde{H})) + \min(w_{\tilde{T}}, w_{\tilde{H}})}{\max(P(\tilde{T}), P(\tilde{H})) + \max(w_{\tilde{T}}, w_{\tilde{H}})}), \quad (5)$$

where

$$P(\tilde{T}) = (\sqrt{(t_1 - t_2)^2 + (w_{\tilde{T}})^2} + (t_3 - t_2) + (\sqrt{(t_4 - t_3)^2 + (w_{\tilde{T}})^2})) + (t_4 - t_1),$$

$$P(\tilde{H}) = (\sqrt{(h_1 - h_2)^2 + (w_{\tilde{H}})^2} + (h_3 - h_2) + (\sqrt{(h_4 - h_3)^2 + (w_{\tilde{H}})^2})) + (h_4 - h_1).$$

Recently, Xu et al. [8] changed the similarity method SCC proposed by Chen and Chen [5], introducing the new similarity S_X . The authors used this method to answer the problems of fuzzy risk analysis.

$$S_X(\tilde{T}, \tilde{H}) = 1 - (\sum_{k=1}^4 \frac{t_k - h_k}{8}) - (\frac{d(\tilde{T}, \tilde{H})}{2}), \quad (6)$$

where $d(\tilde{T}, \tilde{H}) = \frac{(\sqrt{(x_{\tilde{T}}^* - x_{\tilde{H}}^*)^2 + (y_{\tilde{T}}^* - y_{\tilde{H}}^*)^2})}{\sqrt{1.25}}$.

Chen [9] introduced a new proposed method to find a solution

to the similarity problems of GFNs. This method is based on the geometric mean operator. The author used this approach to clear the problems of fuzzy-number used in the information retrieval.

$$S_{SJ}(\tilde{T}, \tilde{H}) = [(\prod_{k=1}^4 (2 - |t_k - h_k|))^{\frac{1}{4}} - 1] \times \frac{\min(y_{\tilde{T}}^*, y_{\tilde{H}}^*)}{\max(y_{\tilde{T}}^*, y_{\tilde{H}}^*)}, \quad (7)$$

where $y_{\tilde{T}}^*$ and $y_{\tilde{H}}^*$ are calculated as in Chen and Chen [5].

3. Approach

We will now present a novel mathematical model, MCESTA (Mohamedou Cheikh Elghotob Cheikh Saad bouh Cheikh Tourad Abass), Abdelmounaim Abdali, a large scale similarity method between GTFNs, call them $\tilde{T}$ and $\tilde{H}$. It is a kind of hybrid of the similarity measures S_k and S_k^q , where each measure is associated with an importance weight (α_k and β_q). MCESTA is an efficient development of the formula FBHSM = $\sum_{k=1}^n \alpha_k \times S_k$ proposed by Gupta et al. [13] to calculate similarity in IR (information retrieval). MCESTA employs a fuzzy inference system (FIS) to find the value of the effective weighting. This hybrid similarity measure is calculated as follows:

$$MCESTA(\tilde{T}, \tilde{H}) = \begin{cases} \sum_{k=1}^n \alpha_k \times S_k(\tilde{T}, \tilde{H}), \\ S_k(\tilde{T}, \tilde{H}) = \sum_{q=1}^{m_k} \beta_q \times S_k^q(\tilde{T}, \tilde{H}), \end{cases}$$

where $\sum_{k=1}^n \alpha_k \leq 1$ and $\sum_{q=1}^{m_k} \beta_q \leq 1$.

Our model will be as follows:

$$MCESTA(\tilde{T}, \tilde{H}) = \sum_{K=1}^n \alpha_K \times [\sum_{q=1}^{m_K} \beta_q S_K^q(\tilde{T}, \tilde{H})], \quad (8)$$

where S_k is a similarity method between $\tilde{T}$ and $\tilde{H}$, S_k^q is a similarity sub-measure between $\tilde{T}$ and $\tilde{H}$ that should take into account the two criteria of their relative location and size. This measure is calculated to determine a reasonable value of similarity S_k . α_k is an importance weight associated with S_k and β_q is an importance weight associated with S_k^q . The number n defines the number of similarity methods S_k used to calculate the model. The number m_k defines the number of similarity sub-measures S_k^q employed to calculate S_k .

3.1 Similarity Measures S_k and S_k^q

The descriptions of the similarity methods employed, the cosine coefficient and the Jaccard Index [14], are given below. This choice of similarity measures will be validated in Section

4. We have $n = 2$, $S_1 = \cos$ and $S_2 = Jaccard$ and we choose $m_1 = 1$ and $m_2 = 2$, which implies, in (9),

$$MCESTA(\tilde{T}, \tilde{H}) = \sum_{k=1}^2 \alpha_k \times [\sum_{q=1}^{m_k} \beta_q S_k^q(\tilde{T}, \tilde{H})].$$

for each k we provide a corresponding m_k .

$$\begin{aligned} MCESTA(\tilde{T}, \tilde{H}) &= \alpha_1 \times [\sum_{q=1}^{m_1} \beta_q S_1^q(\tilde{T}, \tilde{H})] \\ &+ \alpha_2 \times [\sum_{q=1}^{m_2} \beta_q S_2^q(\tilde{T}, \tilde{H})], \\ MCESTA(\tilde{T}, \tilde{H}) &= \alpha_1 \times [\beta_1 S_1^1(\tilde{T}, \tilde{H})] \\ &+ \alpha_2 \times [\beta_1 S_2^1(\tilde{T}, \tilde{H}) + \beta_2 S_2^2(\tilde{T}, \tilde{H})], \\ MCESTA(\tilde{T}, \tilde{H}) &= \alpha_1 \times \beta_1 \cos(\tilde{T}, \tilde{H}) \\ &+ \alpha_2 \times \beta_1 Jaccard_1(\tilde{T}, \tilde{H}) \\ &+ \alpha_2 \times \beta_2 Jaccard_2(\tilde{T}, \tilde{H}), \end{aligned}$$

where $S_1^1 = \cos$, $S_2^1 = Jaccard_1$ and $S_2^2 = Jaccard_2$. To facilitate the calculation of this approach, one needs to make the following changes of variable: $C = \alpha_1 \times \beta_1$, $J_1 = \alpha_2 \times \beta_1$ and $J_2 = \alpha_2 \times \beta_2$. So this equation will become

$$\begin{aligned} MCESTA(\tilde{T}, \tilde{H}) &= C \times \cos(\tilde{T}, \tilde{H}) \\ &+ J_1 \times Jaccard_1(\tilde{T}, \tilde{H}) \\ &+ J_2 \times Jaccard_2(\tilde{T}, \tilde{H}). \end{aligned} \quad (9)$$

3.1.1 The cosine coefficient $\cos(\tilde{T}, \tilde{H})$

The cosine coefficient calculates the similarity between vectors in an easy and more intelligent way: it is by the determination of the direction. $\cos(\tilde{T}, \tilde{H})$ is calculated as follows:

$$x_k = t_k,$$

where $[k = \{1, 2, 3, 4\}]$, $x_5 = w_T$, $x_6 = t_2 - t_1$, $x_7 = t_3 - t_2$, $x_8 = t_4 - t_3$. Also, $y_k = h_k$ where $[k = \{1, 2, 3, 4\}]$, $y_5 = w_H$, $y_6 = h_2 - h_1$, $y_7 = h_3 - h_2$, $y_8 = h_4 - h_3$.

$$\cos(\tilde{T}, \tilde{H}) = \frac{\sum_{k=1}^8 x_k \times y_k}{(\sum_{k=1}^8 x_k^2)^{\frac{1}{2}} \times (\sum_{k=1}^8 y_k^2)^{\frac{1}{2}}}. \quad (10)$$

3.1.2 The Jaccard Index

The Jaccard Index, $Jaccard(\tilde{T}, \tilde{H})$ [10], measures the similarity as follows: the size of the intersection is divided by the size of the union.

Figure 1. The Mamdani for MCESTA.

Figure 2. FIS value membership functions.

$Jaccard_1(\tilde{T}, \tilde{H})$ is calculated as follows: $x_k = t_k - m$, where $[k = \{1, 2, 3, 4\}]$, $x_5 = w_{\tilde{T}}$, $x_6 = t_2 - t_1$, $x_7 = t_3 - t_2$, $x_8 = t_4 - t_3$, and $y_k = h_k - m$, where $[k = \{1, 2, 3, 4\}]$, $y_5 = w_{\tilde{H}}$, $y_6 = h_2 - h_1$, $y_7 = h_3 - h_2$, $y_8 = h_4 - h_3$, where $m = \min(t_1, h_1)$.

$$Jaccard_1(\tilde{T}, \tilde{H}) = \frac{\sum_{k=1}^8 x_k \times y_k}{(\sum_{k=1}^8 x_k^2) + (\sum_{k=1}^8 y_k^2) - \sum_{k=1}^8 x_k \times y_k}. \quad (11)$$

$Jaccard_2(\tilde{T}, \tilde{H})$ is calculated as following: $x_{5-k}^* = M_k - x_k$, where $[k = \{1, 2, 3, 4\}]$, $x_5^* = w_T$, $x_6^* = t_2 - t_1$, $x_7^* = t_3 - t_2$, $x_8^* = t_4 - t_3$, and $y_{5-k}^* = M_k - y_k$, where $[k = \{1, 2, 3, 4\}]$, $y_5^* = w_H$, $y_6^* = h_2 - h_1$, $y_7^* = h_3 - h_2$, $y_8^* = h_4 - h_3$, where $M = \max(t_4, h_4)$.

$$Jaccard_2(\tilde{T}, \tilde{H}) = \frac{\sum_{k=1}^8 x_k^* \times y_k^*}{\sum_{k=1}^8 (x_k^*)^2 + \sum_{k=1}^8 (y_k^*)^2 - \sum_{i=k}^8 x_k^* \times y_k^*}. \quad (12)$$

3.2 The Methods to Define the Weights C , J_1 and J_2

Our choice is based on an FIS, such systems are recognized and have been used in many different fields [15, 16]. To calculate the weights mentioned in Eq. 9, we use the Mamdani-type FIS mentioned in [13, 17–19] adapted to our model. Figure 1 shows the Mamdani for MCESTA.

Figure 3. FIS-diagram for the MCESTA.

Figure 4. FIS-rule surface diagram for C by Cosine and Jaccard1 in MCESTA.

In Table 1 there is presented the FIS fuzzy rule for the MCESTA, which is a set of semantic declarations that define how the FIS-Mamdani must carry out its decision making for input state (Cosine, Jaccard₁, and Jaccard₂) or controlling an output (C , J_1 and J_2). This table uses the values of the functions presented in Figure 2.

In Figure 3, the FIS-diagram for the MCESTA defined as a procedure for developing the association relation (from a given value to an output value) based on the concepts of fuzzy logic. This figure shows the values which are the most important for the weights following C , J_1 and J_2 .

Figure 4 to Figure 9 show the evolution of the FIS-Rule surface diagram for the weights (C , J_1 and J_2) in MCESTA.

3.3 Properties

Table 1. FIS fuzzy rule for MCESTA

IF (Jaccard ₁ is L) and (Jaccard ₂ is L) and (Cos is L) THEN (J ₁ is L) (J ₂ is L) (C is L) (1).
IF (Jaccard ₁ is L) and (Jaccard ₂ is M) and (Cos is L) THEN (J ₁ is L) (J ₂ is M) (C is L) (1).
IF (Jaccard ₁ is L) and (Jaccard ₂ is H) and (Cos is L) THEN (J ₁ is L) (J ₂ is H) (C is L) (1).
IF (Jaccard ₁ is M) and (Jaccard ₂ is L) and (Cos is L) THEN (J ₁ is M) (J ₂ is L) (C is L) (1).
IF (Jaccard ₁ is M) and (Jaccard ₂ is M) and (Cos is L) THEN (J ₁ is M) (J ₂ is M) (C is L) (1).
IF (Jaccard ₁ is M) and (Jaccard ₂ is H) and (Cos is L) THEN (J ₁ is M) (J ₂ is H) (C is L) (1).
IF (Jaccard ₁ is H) and (Jaccard ₂ is L) and (Cos is L) THEN J ₁ is H) (J ₂ is L) (C is L) (1).
IF (Jaccard ₁ is H) and (Jaccard ₂ is M) and (Cos is L) THEN (J ₁ is H) (J ₂ is M) (C is L) (1).
IF (Jaccard ₁ is H) and (Jaccard ₂ is H) and (Cos is L) THEN (J ₁ is H) (J ₂ is H) (C is L) (1).
IF (Jaccard ₁ is L) and (Jaccard ₂ is L) and (Cos is M) THEN (J ₁ is L) (J ₂ is L) (C is M) (1).
IF (Jaccard ₁ is L) and (Jaccard ₂ is M) and (Cos is M) THEN (J ₁ is L) (J ₂ is M) (C is M) (1).
IF (Jaccard ₁ is L) and (Jaccard ₂ is H) and (Cos is M) THEN (J ₁ is L) (J ₂ is H) (C is M) (1).
IF (Jaccard ₁ is M) and (Jaccard ₂ is L) and (Cos is M) THEN (J ₁ is M) (J ₂ is L) (C is M) (1).
IF (Jaccard ₁ is M) and (Jaccard ₂ is M) and (Cos is M) THEN (J ₁ is M) (J ₂ is M) (C is M) (1).
IF (Jaccard ₁ is M) and (Jaccard ₂ is H) and (Cos is M) THEN (J ₁ is M) (J ₂ is H) (C is M) (1).
IF (Jaccard ₁ is H) and (Jaccard ₂ is L) and (Cos is M) THEN (J ₁ is H) (J ₂ is L) (C is M) (1).
IF (Jaccard ₁ is H) and (Jaccard ₂ is M) and (Cos is M) THEN (J ₁ is H) (J ₂ is M) (C is M) (1).
IF (Jaccard ₁ is H) and (Jaccard ₂ is H) and (Cos is M) THEN (J ₁ is H) (J ₂ is H) (C is M) (1).
IF (Jaccard ₁ is L) and (Jaccard ₂ is L) and (Cos is H) THEN (J ₁ is L) (J ₂ is L) (C is H) (1).
IF (Jaccard ₁ is L) and (Jaccard ₂ is M) and (Cos is H) THEN (J ₁ is L) (J ₂ is M) (C is H) (1).
IF (Jaccard ₁ is L) and (Jaccard ₂ is H) and (Cos is H) THEN (J ₁ is L) (J ₂ is H) (C is H) (1).
IF (Jaccard ₁ is M) and (Jaccard ₂ is L) and (Cos is H) THEN (J ₁ is M) (J ₂ is L) (C is H) (1).
IF (Jaccard ₁ is M) and (Jaccard ₂ is M) and (Cos is H) THEN (J ₁ is M) (J ₂ is M) (C is H) (1).
IF (Jaccard ₁ is M) and (Jaccard ₂ is H) and (Cos is H) THEN (J ₁ is M) (J ₂ is H) (C is H) (1).
IF (Jaccard ₁ is H) and (Jaccard ₂ is L) and (Cos is H) THEN (J ₁ is H) (J ₂ is L) (C is H) (1).
IF (Jaccard ₁ is H) and (Jaccard ₂ is M) and (Cos is H) THEN (J ₁ is H) (J ₂ is M) (C is H) (1).
IF (Jaccard ₁ is H) and (Jaccard ₂ is H) and (Cos is H) THEN (J ₁ is H) (J ₂ is H) (C is H) (1).

L, Low; M, Medium; H, High.

3.3.1 Identical

$$\tilde{T} = \tilde{H} \Leftrightarrow MCESTA(\tilde{T}, \tilde{H}) = 1.$$

3.3.2 Symmetric relation

$$MCESTA(\tilde{T}, \tilde{H}) = MCESTA(\tilde{H}, \tilde{T}).$$

3.3.3 Scale-free

This property has been used by Hwang and Yang [10] for validating their proposed similarity method. We have that

$$MCESTA(\tilde{T}_1, \tilde{H}_1) = MCESTA(\tilde{T}, \tilde{H})$$

for $\tilde{T}_1 = \alpha \times \tilde{T} = (\alpha \times t_1, \alpha \times t_2, \alpha \times t_3, \alpha \times t_4, \alpha \times w_{\tilde{T}})$ and $\tilde{H}_1 = \alpha \times \tilde{H} = (\alpha \times h_1, \alpha \times h_2, \alpha \times h_3, \alpha \times h_4, \alpha \times w_{\tilde{H}})$, where $\alpha \geq 0$. Normally, the best similarity should be scale free. This is a very significant property for the large scale. According to this result, we deduce that a very large data $Set_1 = (\tilde{T}_1, \tilde{H}_1)$ can be analyzed using another batch of data of small size $Set = (\tilde{T}, \tilde{H})$. But according to the scale relation $\tilde{T}_1 = \alpha \times \tilde{T}$ and $\tilde{H}_1 = \alpha \times \tilde{H}$, the interpretation and the analysis of Set gives a global idea of the interpretation of Set_1 , which is important when it can be difficult to treat everything in the context of big data or the machines don't have sufficient capacity. This property is very important for large scale studies.

Figure 5. FIS-rule surface diagram for C by Cosine and Jaccard2 in MCESTA.

Figure 8. FIS-rule surface diagram for J2 by Jaccard2 and Cosine in MCESTA.

Figure 6. FIS-rule surface diagram for J1 by Jaccard1 and Cosine in MCESTA.

Figure 9. FIS-rule surface diagram for J2 by Jaccard2 and Jaccard2 in MCESTA.

Figure 7. FIS-rule surface diagram for J1 by Jaccard1 and Jaccard2 in MCESTA.

3.4 Proof of Properties

To see that our model verifies the three properties, it is enough that every similarity (Cosine, Jaccard₁ and Jaccard₂) used in this model satisfies these properties. For the sub-measures Jaccard₁ and Jaccard₂, this is proved by Hwang and Yang [10]. It remains to treat Cosine.

3.4.1 MCESTA is identical

$$\tilde{T} = \tilde{H} \Leftrightarrow \cos(\tilde{T}, \tilde{H}) = 1.$$

(i) The $\Rightarrow$ is proved by observing that since $\tilde{T} = \tilde{H}$, then $x_k = y_k, k = \{1, 2, 3, 4, 6, 7, 8\}$. Therefore

$$\frac{\sum_{k=1}^8 x_k \times y_k}{(\sum_{k=1}^8 x_k^2)^{\frac{1}{2}} \times (\sum_{k=1}^8 y_k^2)^{\frac{1}{2}}} = \frac{\sum_{k=1}^8 x_k \times x_k}{(\sum_{k=1}^8 x_k^2)^{\frac{1}{2}} \times (\sum_{k=1}^8 x_k^2)^{\frac{1}{2}}}$$

$$= \frac{\sum_{k=1}^8 x_k^2}{\sum_{k=1}^8 x_k^2} = 1.$$

Thus ,

$$\tilde{t}ilde{T} = \tilde{H} \Rightarrow \cos(\tilde{T}, \tilde{H}) = 1.$$

(ii) The $\Leftarrow$ is proved as follows: Since

$$\begin{aligned} \cos(\tilde{T}, \tilde{H}) = 1 &\Rightarrow \frac{\sum_{k=1}^8 (x_k \times y_k)}{(\sum_{k=1}^8 x_k^2)^{\frac{1}{2}} \times (\sum_{k=1}^8 y_k^2)^{\frac{1}{2}}} = 1, \\ \sum_{k=1}^8 (x_k \times y_k) &= (\sum_{k=1}^8 x_k^2)^{\frac{1}{2}} \times (\sum_{k=1}^8 y_k^2)^{\frac{1}{2}} \\ &\Rightarrow (\sum_{k=1}^8 x_k \times y_k)^2 = (\sum_{k=1}^8 x_k^2) \times (\sum_{k=1}^8 y_k^2), \end{aligned}$$

we have that:

$$\begin{aligned} &(\sum_{k=1}^8 x_k^2) \times (\sum_{k=1}^8 y_k^2) \\ &= \sum_{k=1}^8 x_k^2 \times y_k^2 + (\sum_{k=1}^8 x_k^2) \times (\sum_{q=1, q \neq k}^8 y_q^2) \\ &= \sum_{k=1}^8 x_k^2 \times y_k^2 + \sum_{k=1}^8 \sum_{q=1, q \neq k}^8 x_k^2 \times y_q^2, \end{aligned}$$

$$(a) (\sum_{k=1}^8 x_k^2) \times (\sum_{k=1}^8 y_k^2) = \sum_{k=1}^8 x_k^2 \times y_k^2 + \sum_{k=1}^8 \sum_{q=1, q \neq k}^8 x_k^2 \times y_q^2,$$

and we have that :

$$(b) (\sum_{k=1}^8 x_k \times y_k)^2 = \sum_{k=1}^8 x_k^2 \times y_k^2 + \sum_{k=1}^8 \sum_{q=1, q \neq k}^8 (x_k \times y_k) \times (x_q \times y_q),$$

($x_q \times y_q$), the intersection between (a) and (b) is 0:

$$\begin{aligned} &\sum_{k=1}^8 \sum_{q=1, q \neq k}^8 x_k^2 \times y_k^2 - \sum_{k=1}^8 \sum_{q=1, q \neq k}^8 (x_k \times y_k) \times (x_q \times y_q) \\ &= 0, \\ &(\sum_{k=1}^8 \sum_{q=1, q \neq k}^8 (x_k \times y_q)) \times [(x_k \times y_q) - (x_q \times y_k)] = 0, \end{aligned}$$

we have :

$$\begin{aligned} 0 &\leq \frac{\sum_{k=1}^8 x_k \times y_k}{(\sum_{k=1}^8 x_k^2)^{\frac{1}{2}} \times (\sum_{k=1}^8 y_k^2)^{\frac{1}{2}}} \leq 1 \\ &\Rightarrow 0 \leq \frac{(\sum_{k=1}^8 x_k \times y_k)^2}{(\sum_{k=1}^8 x_k^2) \times (\sum_{k=1}^8 y_k^2)} \leq 1 \\ &\Rightarrow (a) - (b) \geq 0, \end{aligned}$$

so (a)–(b)= 0 $\Rightarrow (x_k \times y_q)) \times [(x_k \times y_q) - (x_q \times y_k)] = 0$, we have $x_k \neq 0$ and $y_k \neq 0$. Thus (a)–(b)= 0 $\Rightarrow [(x_k \times y_q) - (x_q \times y_k)] = 0. \Rightarrow \exists \lambda / \forall \lambda \geq 0, \frac{x_k}{y_k} = \frac{x_q}{y_q} = \lambda$. particularly $\lambda = 1 \Rightarrow x_k = y_k$.

Therefore, we have that $x_k = y_k, [k = 1, 2, 3, 4, 5, 6, 7, 8]$, $\tilde{T} = \tilde{H}$.

3.4.2 MCESTA is a symmetric relation

$$\cos(\tilde{T}, \tilde{H}) = \cos(\tilde{H}, \tilde{T}).$$

This follows since $\sum_{k=1}^8 x_k \times y_k = \sum_{k=1}^8 y_k \times x_k$ and $(\sum_{k=1}^8 x_k^2)^{\frac{1}{2}} \times (\sum_{k=1}^8 y_k^2)^{\frac{1}{2}} = (\sum_{k=1}^8 y_k^2)^{\frac{1}{2}} \times (\sum_{k=1}^8 x_k^2)^{\frac{1}{2}}$. It simple to show that $\cos(\tilde{T}, \tilde{H}) = \cos(\tilde{H}, \tilde{T})$.

3.4.3 MCESTA is scale-free

$$\cos(\tilde{T}_1, \tilde{H}_1) = \cos(\tilde{T}, \tilde{H}).$$

Suppose $\tilde{T}_1 = \alpha \times \tilde{T} = (\alpha \times t_1, \alpha \times t_2, \alpha \times t_3, \alpha \times t_4, \alpha \times w_{\tilde{T}})$ and $\tilde{H}_1 = \alpha \times \tilde{H} = (\alpha \times h_1, \alpha \times h_2, \alpha \times h_3, \alpha \times h_4, \alpha \times w_{\tilde{H}})$, where $\alpha \geq 0$.

We have $\cos(\tilde{T}_1, \tilde{H}_1) = \frac{\sum_{k=1}^8 (\alpha \times x_k) \times (\alpha \times y_k)}{(\sum_{k=1}^8 ((\alpha \times x_k)^2)^{\frac{1}{2}} \times (\sum_{k=1}^8 (\alpha \times y_k)^2)^{\frac{1}{2}}} = \frac{\sum_{k=1}^8 x_k \times y_k}{(\sum_{k=1}^8 x_k^2)^{\frac{1}{2}} \times (\sum_{k=1}^8 y_k^2)^{\frac{1}{2}}} = \cos(\tilde{T}, \tilde{H})$.

4. A Comparative Study

4.1 Implementation

Our approach to similarity and the existing methods S_C (Eq. 1), S_{HC} (Eq. 2), S_L (Eq. 3), S_{CC} (Eq. 4), S_{WC} (Eq. 5), S_X (Eq. 6), S_{SJ} (Eq. 7) are implemented employing the Python 3.6 with libraries data science (NumPy, SciPy). All calculations were done with a quad-core processor 3.60 GHz and a 16 GB memory for the JVM. The sets of GTFNs are implemented as associated lists of arrays (Array List).

4.2 Examples and Comparisons

To validate and compare our contributions with the existing methods, we made the calculations on the sets already used by Hwang and Yang [10] in 2014. There are 21 sets (Set₁ to Set₂₁) of fuzzy numbers, shown in Figure 10 to Figure 30 respectively. The estimated times taken by our approach and by the existing

Figure 10. Set₁.

Figure 14. Set₅.

Figure 11. Set₂.

Figure 15. Set₆.

Figure 12. Set₃.

Figure 16. Set₇.

Figure 13. Set₄.

Figure 17. Set₈.

Figure 18. Set₉.

methods are given in Table 2. We can demonstrate and analyze the weaknesses and limitations of each of the existing similarity measures from the table.

We can analyze Table 2 in terms of three types: incorrect results, scale-dependent results, and direction.

4.2.1 Incorrect results

- We have in Set₁, $S_{HC}(\tilde{A}, \tilde{A}_1) = 1$: we can judge the result as incorrect.

Table 2. Comparison

Method / Set _i	S_L	S_{HC}	S_C	S_{CC}	S_{WC}	S_X	S_{SJ}	Our approach
1	0.9167	1.0	0.975	0.8357	0.95	0.9627	0.8356	0.9877
2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
3	0.5	0.7692	0.7	0.42	0.682	0.7136	0.5997	0.8475
4	0.5	0.7692	0.7	0.49	0.7	0.7158	0.7	0.8551
5	1.0	1.0	1.0	0.8	0.8248	0.9652	0.8	0.9797
6	#	1.0	1.0	1.0	1.0	1.0	1.0	1.0
7	0.0	0.9091	0.9	0.9	0.9	0.9053	0.9	0.9725
8	0.5	0.9091	0.9	0.54	0.8411	0.8631	0.5991	0.9482
9	0.6667	0.9091	0.9	0.81	0.9	0.9053	0.9	0.9756
10	0.8333	1.0	0.9	0.9	0.7833	0.95	0.8974	0.9311
11	0.75	1.0	0.9	0.72	0.8003	0.9127	0.72	0.9572
12	0.8	0.9375	0.9	0.78	0.8309	0.8904	0.8959	0.9068
13	0.75	0.9091	0.9	0.81	0.9	0.9053	0.9	0.979
14	1.0	1.0	1.0	0.7	0.7209	0.9553	0.7	0.9484
15	0.75	1.0	0.95	0.9048	0.6215	0.9675	0.9042	0.9187
16	0.4	0.7692	0.7	0.49	0.6222	0.7158	0.6971	0.8125
17	0.25	0.7692	0.7	0.49	0.7	0.7158	0.7	0.828
18	0.5	0.7692	0.7	0.49	0.7	0.7158	0.7	0.8551
19	0.5	0.7692	0.7	0.49	0.7	0.7158	0.7	0.7636
20	0.5	0.7692	0.7	0.49	0.7	0.7158	0.7	0.8551
21	0.81	0.8696	0.85	0.7225	0.85	0.8579	0.85	0.8551

Bold text represents incorrect results and italicized text, scale-dependent results.

Figure 19. Set₁₀. Figure 21. Set₄. Figure 20. Set₁₁. Figure 22. Set₁₃.

Figure 23. Set₁₄.

Figure 28. Set₁₉.

Figure 24. Set₁₅.

Figure 29. Set₂₀.

Figure 25. Set₁₆.

Figure 30. Set₂₁.

Figure 26. Set₁₇.

Figure 27. Set₁₈.

- For Set₃ and Set₄, we have that the similarity of Set₄

should be more similar than that of Set₃, but the similarity methods $S_C(\tilde{A}, \tilde{A}_1)$, $S_{HC}(\tilde{A}, \tilde{A}_1)$ and $S_L(\tilde{A}, \tilde{A}_1)$ produce the same result: we can judge the result as incorrect.

- We have in Set₅, $\tilde{A} \neq \tilde{A}_1$. The similarity produced by $S_C(\tilde{A}, \tilde{A}_1)$, $S_C(\tilde{A}, \tilde{A}_1)$ and $S_L(\tilde{A}, \tilde{A}_1)$ is 1: we can say the result is incorrect.
- We have in Set₆ for $S_L(\tilde{A}, \tilde{A}_1)$, it can't evaluate the similarity degree. we can judge the result as incorrect.
- For Set₇, the similarity produced by $S_L(\tilde{A}, \tilde{A}_1)$ is 0: the result is incorrect.
- For Set₈ and Set₉, we have that the similarity of Set₄ should be different from the similarity of those with Set₃, but the similarity methods $S_C(\tilde{A}, \tilde{A}_1)$ and $S_C(\tilde{A}, \tilde{A}_1)$ produce the same result: we can say the result is incorrect.

- For Set₉ and Set₁₀, we have that the similarity of Set₉ should be more similar than that Set₁₀, however, Table 2 proves that the similarity produced by the methods $S_C(\tilde{A}, \tilde{A}_1)$, $S_{HC}(\tilde{A}, \tilde{A}_1)$, $S_L(\tilde{A}, \tilde{A}_1)$, $S_{CC}(\tilde{A}, \tilde{A}_1)$ and $S_X(\tilde{A}, \tilde{A}_1)$ are incorrect results.
- We have in Set₁₁, $\tilde{A} \neq \tilde{A}_1$. The similarity produced by $S_{HC}(\tilde{A}, \tilde{A}_1)$ is 1: we can judge the result as incorrect.
- We have in Set₁₄, $\tilde{A} \neq \tilde{A}_1$. The similarity produced by $S_C(\tilde{A}, \tilde{A}_1)$, $S_{HC}(\tilde{A}, \tilde{A}_1)$ and $S_L(\tilde{A}, \tilde{A}_1)$ is 1: the result is incorrect.
- For Set₁₄ and Set₁₅, we have that the similarity of Set₁₄ should be more similar than that of Set₁₅. However, Table 2 proves that the similarity produced by the methods $S_C(\tilde{A}, \tilde{A}_1)$, $S_{HC}(\tilde{A}, \tilde{A}_1)$, $S_L(\tilde{A}, \tilde{A}_1)$, $S_{CC}(\tilde{A}, \tilde{A}_1)$, $S_X(\tilde{A}, \tilde{A}_1)$ and $S_{SJ}(\tilde{A}, \tilde{A}_1)$ are incorrect results.
- It can be seen for the two different Set₁₆ and Set₁₇ that the methods $S_C(\tilde{A}, \tilde{A}_1)$, $S_{HC}(\tilde{A}, \tilde{A}_1)$, $S_{CC}(\tilde{A}, \tilde{A}_1)$ and $S_X(\tilde{A}, \tilde{A}_1)$ produce incorrect results.
- For Set₁₈ and Set₁₉, we have that the similarity of Set₁₈ should be more similar than that of Set₁₉, but the similarity methods $S_C(\tilde{A}, \tilde{A}_1)$, $S_{HC}(\tilde{A}, \tilde{A}_1)$, $S_L(\tilde{A}, \tilde{A}_1)$, $S_{CC}(\tilde{A}, \tilde{A}_1)$, $S_{WC}(\tilde{A}, \tilde{A}_1)$, $S_X(\tilde{A}, \tilde{A}_1)$ and $S_{SJ}(\tilde{A}, \tilde{A}_1)$ produce the same result: we can say the result is incorrect.

4.2.2 Scale-dependent results

- For Set₂₀ and Set₂₁, we have that Set₂₀ and Set₂₁ are in double-scale relation. Usually, the best similarity methods must verify property 3 (scale-free), but the similarity methods $S_C(\tilde{A}, \tilde{A}_1)$, $S_{HC}(\tilde{A}, \tilde{A}_1)$, $S_L(\tilde{A}, \tilde{A}_1)$, $S_{CC}(\tilde{A}, \tilde{A}_1)$, $S_{WC}(\tilde{A}, \tilde{A}_1)$, $S_X(\tilde{A}, \tilde{A}_1)$ and $S_{SJ}(\tilde{A}, \tilde{A}_1)$ are scale-dependent (are not scale-free), while our approach is scale-free.

4.2.3 Direction(Cosine) passing to large-scale

- The direction similarity is very important. This measurement is used in the big data framework when the data tends towards the infinite. But the similarity methods $S_C(\tilde{A}, \tilde{A}_1)$, $S_{HC}(\tilde{A}, \tilde{A}_1)$, $S_L(\tilde{A}, \tilde{A}_1)$, $S_{CC}(\tilde{A}, \tilde{A}_1)$, $S_{WC}(\tilde{A}, \tilde{A}_1)$, $S_X(\tilde{A}, \tilde{A}_1)$ and $S_{SJ}(\tilde{A}, \tilde{A}_1)$ do not use the direction, while our approach uses this measurement with a weight $C = 0.508$, see Figure 3.

After the analysis of Table 2, the three properties and the direction based on the cosine coefficient help this similarity approach to be the best choice for the large scale.

5. Conclusion

A novel mathematical model MCESTA for GTFNs has been presented (as well as the existing methods). This model is based on using weights associated with each one of several difference nt similarity measures. We have been able to infer the importance weights by a Mamdani-type FIS [17]. This model uses the cosine coefficient and Jaccard Index. Three properties of the model are proved, one property is advantageous for being used with large scale datasets. A comparative study has also been presented to explain how onovel approach can overcome the limitations and weaknesses of the existing methods. This approach will help us develop an intelligent filtering of the pub-sub system [20, 21].

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

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