On Soft Generalized Closed Sets in a Soft Topological Space with a Soft Weak Structure

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Abstract

In this work, we introduce the notion of generalized ω -closed set in a soft topological space with a soft weak structure. And some basic properties of this new class are investigated by using the concept of weak structure. Moreover, we study soft ω - $T_{\frac{1}{2}}$ -spaces defined by soft $g\omega$ -closed sets and study some properties of it by using $g\omega$ -open sets.

Keywords: Soft set, Soft topology, Soft weak structure, Soft generalized closed set

1. Introduction

As a solution to many problems, scientists have resorted to use approximate solution. In 1999, Molodtsova [1] initiated the concept of soft set theory as a new mathematical tool which is free from the problems mentioned above. Later on Maji et al [2] proposed several operations on soft sets and some basic properties and then Pei and Miao [3] investigated the relationships between soft sets and information systems.

In 2011, Shabir and Naz [4] introduced the notion of soft topological spaces and Min [5] corrected some their results. Zorlutuna et al. [6] continued to study the properties of soft topological spaces by defining the concepts of interior, soft neighborhoods in soft topological spaces. Varo and Aygun [7] presented soft Hausdorff spaces and introduced some new concepts such as convergence of sequences. Levine [8] introduced generalized closed sets in topological spaces.

In 2013, Cagman et al. [9] defined soft topological spaces by modifying the soft set. Also, Roy and Samanta [10] strengthen the definition of the soft topological spaces presented in [9] and they used the base and the subbase to characterize its properties.

In 1997, Császár [4] has studied generalized topological notions in collections which are closed under unions. Many other authors [12–15] investigated the properties of the generalized topology. Recently, Császár [16] introduced a new notion called weak structures. Let X be a non-empty set and $P(X)$ be its power set. A structure ω on X is called a weak structure (briefly, WS) on X if and only if $\phi \in \omega$ [16].

In 2012, Al-Omari and Noiri [17] introduced and studied a kind of sets called generalized ω -closed (briefly, $g\omega$ -closed) sets in topological space. In this paper, we introduce the notions of soft generalized ω -closed (briefly, soft $g\omega$ -closed) sets and soft $g\omega$ -open sets in soft topological spaces. Also, we will investigate some new soft separation axioms. In particular, we study soft ω - $T_{\frac{1}{2}}$ -spaces.

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Let X be a non-empty set, E a set of parameters, $P(X)$ denote the power set of X and A be a non-empty subset of E .

Definition 1.1 ([1]). For $A \subseteq E$, a pair (F, A) is called a soft set over X , where F is a mapping given by $F : A \rightarrow P(X)$. For $e \in A$, $F(e)$ may be considered as the set of e -approximate elements of the soft set (F, A) .

Definition 1.2 ([5]). A soft set (F, A) over X is said to be:

1. A null soft set denoted by $\tilde{\emptyset}$ if $F(e) = \emptyset$ for all $e \in A$.
2. An absolute soft set denoted by $\tilde{X}$ if $F(e) = X$ for all $e \in A$.

Definition 1.3 ([2, 5, 18]). For any two soft sets (F, A) and (G, B) defined over a common universe X , we have:

1. $(F, A) \tilde{\subset} (G, B)$ iff $A \subseteq B$ and $F(e) \subseteq G(e)$ for all $e \in A$.
2. $(F, A) \tilde{=} (G, B)$ iff $(F, A) \tilde{\subset} (G, B)$ and $(G, B) \tilde{\subset} (F, A)$.
3. $(F, A) \tilde{\cup} (G, B) \tilde{=} (H, C)$ where $C = A \cup B$ and

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \cup G(e), & \text{if } e \in A \cap B. \end{cases}$$

for all $e \in C$.

4. $(F, A) \tilde{\cap} (G, B) \tilde{=} (K, D)$ where $D = A \cap B$ and $K(e) = F(e) \cap G(e)$ for all $e \in C$.
5. $x \in (F, A)$ where $x \in X$ iff $x \in F(e)$ for all $e \in A$ and $x \notin (F, A)$ whenever $x \notin F(e)$ for some $e \in A$.
- (6) $(F, E) \tilde{-} (G, E) \tilde{=} (M, E)$ where $M(e) = F(e) - G(e)$ for all $e \in E$.

Definition 1.4 ([19]). For a soft set (F, A) over X , the relative complement of (F, A) (denoted by $(F, A)'$) is defined by $(F, A)' \tilde{=} (F', A)$, where $F' : A \rightarrow P(X)$ is given by $F'(e) = X - F(e)$ for all $e \in A$.

Definition 1.5 ([4]). Let τ be the collection of soft sets over X , then τ is called a soft topology on X if τ satisfies the following axioms:

1. $\tilde{\emptyset}, \tilde{X}$ belong to τ .
2. The union of any number of soft sets in τ belong to τ .

3. The intersection of any two soft sets in τ belong to τ .

The triple (X, τ, E) is called a soft topological space over X . The member of τ are said to be soft open set in X . A soft set (F, E) over X is said to be soft closed in X if its relative complement $(F, E)'$ belong to τ .

Lemma 1.6 ([14]). Let (F, E) be a soft set over X and $x \in X$, then

1. $x \in (F, E)$ iff $(x, E) \tilde{\subset} (F, E)$
2. if $(x, E) \cap (F, E) = \emptyset$, then $x \notin (F, E)$.

Csaszar defined $i_\omega(A)$ as the union of all ω -open subsets of A and $c_\omega(A)$ as the intersection of all ω -closed sets containing A .

Let X be a non-empty set and E be a set of parameters. A collection ω of soft sets defined over X with respect to E is called a soft weak structure [20] iff $\tilde{\emptyset} \in \omega$. A soft set (F, A) is called ω -open soft set iff $(F, A) \in \omega$ and called ω -closed soft set iff $(F, A)' \in \omega$.

Definition 1.7 ([19]). In a soft topological spaces (X, τ, E) , a soft set (F, E) over X is called a soft generalized closed set (briefly, soft g-closed) if $Cl(F, E) \subseteq (G, E)$ whenever $(F, E) \subseteq (G, E)$ and (G, E) is soft open in (X, τ, E) .

Lemma 1.8 ([20]). Let ω be a soft weak structure defined over X with respect to E and $(F, E), (G, E)$ are two soft sets over X , then:

1. $i_\omega(F, E) \tilde{\subset} (F, E)$
2. If $(F, E) \tilde{\subset} (G, E)$, then $i_\omega(F, E) \tilde{\subset} i_\omega(G, E)$ and $c_\omega(F, E) \tilde{\supset} c_\omega(G, E)$
3. $i_\omega(i_\omega(F, E)) \tilde{=} i_\omega(F, E)$ and $c_\omega(c_\omega(F, E)) \tilde{=} c_\omega(F, E)$.
4. $i_\omega(F, E)' = (c_\omega(F, E))'$ and $c_\omega(F, E)' = (i_\omega(F, E))'$.

Lemma 1.9 ([20]). Let ω be a soft weak structure defined over X with respect to the parameters set E and (F, E) be a soft set, then:

1. $x \in i_\omega(F, E)$ if there exists an ω -open soft set (G, E) such that $x \in (G, E) \tilde{\subset} (F, E)$.
2. $x \in c_\omega(F, E)$ if and only if $(G, E) \tilde{\cap} (F, E) \neq \tilde{\emptyset}$ for all $(G, E) \in \omega$ such that $x \in (G, E)$.
3. If $(F, E) \in \omega$, then $(F, E) = i_\omega(F, E)$ and if (F, E) is ω -closed soft set, then $(F, E) = c_\omega(F, E)$

Lemma 1.10 ([4]). Let (X, τ, E) be a soft topological space over X . If (x, E) is a soft closed set in τ for each $x \in X$, then (X, τ, E) is a soft T_1 -space.

Definition 1.11 ([20]). A soft topological space (X, τ, E) is called:

1. Soft ω - T_0 if for each $x, y \in X$ such that $x \neq y$, there exists a soft ω -open set (F, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $x \notin (F, E)$ and $y \in (F, E)$.
2. Soft ω - T_1 if for each $x, y \in X$ such that $x \neq y$, there exists soft ω -open sets (F, E) and (G, E) such that $x \in (F, E)$, $y \notin (F, E)$, $x \notin (G, E)$ and $y \in (G, E)$.

2. Soft $g\omega$ -Closed Set

Definition 2.1. Let X be a non-empty set and E be a set of parameters. Let a collection ω be a soft weak structure on a soft topological space (X, τ, E) . Then a soft set (F, E) over X is called a soft generalized ω -closed set (briefly, soft $g\omega$ -closed set) if $c_\omega(F, E) \tilde{\subset} (G, E)$ whenever $(F, E) \tilde{\subset} (G, E)$ and (G, E) is soft open in (X, τ, E) . The complement of a soft $g\omega$ -closed set is called a soft generalized ω -open (briefly, soft $g\omega$ -open) set.

Example 2.2. Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$ where

$$\begin{aligned} F_1(e_1) &= \{h_2\}, & F_1(e_2) &= \{h_1\}; \\ F_2(e_1) &= \{h_2, h_3\}, & F_2(e_2) &= \{h_1, h_2\}; \\ F_3(e_1) &= \{h_1, h_2\}, & F_3(e_2) &= \{h_1, h_3\}; \\ F_4(e_1) &= \{h_1\}, & F_4(e_2) &= \{h_3\}. \end{aligned}$$

Let $\omega = \{\tilde{\emptyset}, (F_1, E), (F_3, E)\}$ be a soft weak structure over X with respect to E . Then (G, E) is a soft $g\omega$ -closed defined by $G(e_1) = \tilde{\emptyset}$; $G(e_2) = \{h_2\}$, but (F_1, E) is not soft $g\omega$ -closed.

Remark 2.3. Let ω be a soft weak structure on a soft topological space (X, τ, E) . Then every soft $g\omega$ -closed set is soft g -closed. The following example shows that the converse need not be true in general.

Example 2.4. In Example 2.2, (F_2, E) is soft g -closed in (X, τ, E) but not soft $g\omega$ -closed over X .

Remark 2.5. For a soft weak structure ω on a soft topological space over X , every ω -closed set is a soft $g\omega$ -closed set. In fact, if (F, E) is a ω -closed set $(F, E) \tilde{\subset} (G, E)$ and (G, E) is soft open, then $(F, E) = c_\omega(F, E) \tilde{\subset} (G, E)$, so that (F, E) is

soft $g\omega$ -closed. The following example shows that the converse need not be true in general.

Example 2.6. In Example 2.2, (G, E) is soft $g\omega$ -closed but not ω -closed.

Theorem 2.7. Let ω be a soft weak structure on a soft topological space (X, τ, E) . If (F, E) is soft $g\omega$ -closed in X and $(F, E) \tilde{\subset} (H, E) \tilde{\subset} c_\omega(F, E)$, then (H, E) is soft $g\omega$ -closed

Proof. Suppose that (F, E) is soft $g\omega$ -closed over X and $(F, E) \tilde{\subset} (H, E) \tilde{\subset} c_\omega(F, E)$. Let $(H, E) \tilde{\subset} (G, E)$ and (G, E) is soft open in X . Since $(F, E) \tilde{\subset} (H, E)$ and $(H, E) \tilde{\subset} (G, E)$, we have $(F, E) \tilde{\subset} (G, E)$. Since (F, E) is soft $g\omega$ -closed, then $c_\omega(F, E) \tilde{\subset} (G, E)$. Since $(H, E) \tilde{\subset} c_\omega(F, E)$, we have $c_\omega(H, E) \tilde{\subset} c_\omega(F, E) \tilde{\subset} (G, E)$. Therefore (H, E) is soft $g\omega$ -closed. $\square$

The next example shows that the intersection of two soft $g\omega$ -closed sets is not in general soft $g\omega$ -closed.

Example 2.8. Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$ where

$$\begin{aligned} F_1(e_1) &= \{h_1\}, & F_1(e_2) &= \{h_2\}; \\ F_2(e_1) &= \{h_3\}, & F_2(e_2) &= \{h_1, h_3\}; \\ F_3(e_1) &= \{h_1, h_2\}, & F_3(e_2) &= \{h_2, h_3\}; \\ F_4(e_1) &= \{h_1, h_3\}, & F_4(e_2) &= \emptyset. \end{aligned}$$

Let $\omega = \{\tilde{\emptyset}, (F_2, E)\}$ be a soft weak structure over X with respect to E . Then (H, E) be a soft $g\omega$ -closed defined by $H(e_1) = H(e_2) = \{h_2\}$. Thus it can be easily checked that $(F_1, E) \tilde{\cap} (H, E)$ is not a soft $g\omega$ -closed set.

Theorem 2.9. Let ω be a soft weak structure on a soft topological space (X, τ, E) . If (F, E) is a soft $g\omega$ -closed set, then $c_\omega(F, E) - (F, E)$ does not contain any non-empty soft closed set.

Proof. Let (H, E) be a soft closed subset of X such that $(H, E) \tilde{\subset} c_\omega(F, E) - (F, E)$, where (F, E) is soft $g\omega$ -closed. Since $(H, E)'$ is soft open, $(F, E) \tilde{\subset} (H, E)'$ and (F, E) is soft $g\omega$ -closed, $c_\omega(F, E) \tilde{\subset} (H, E)'$ and thus $(H, E) \tilde{\subset} [c_\omega(F, E)]'$. Thus $(H, E) \tilde{\subset} [c_\omega(F, E)]' \tilde{\cap} c_\omega(F, E) = \tilde{\emptyset}$ and hence $(H, E) = \tilde{\emptyset}$. $\square$

If $c_\omega(F, E) - (F, E)$ does not contain any non-empty soft closed subset of X , then (F, E) need not be soft $g\omega$ -closed in general.

Example 2.10. In Example 2.8, let (G, E) be a soft set defined by $G(e_1) = \tilde{\emptyset}$, $G(e_2) = \{h_2\}$. Then $c_\omega(G, E) - (G, E) =$

(K, E) does not contain any non empty soft closed set such that $K(e_1) = \{h_1, h_2\}$, $K(e_2) = \emptyset$, but (G, E) is not a soft $g\omega$ -closed set.

Corollary 2.11. Let ω be a soft weak structure on a soft topological space (X, τ, E) and (F, E) be a soft $g\omega$ -closed set. Then $c_\omega(F, E) = (F, E)$ if and only if $c_\omega(F, E) - (F, E)$ is soft closed.

Proof. Let (F, E) be a soft $g\omega$ -closed set. If $c_\omega(F, E) = (F, E)$, then $c_\omega(F, E) - (F, E) = \emptyset$, and $c_\omega(F, E) - (F, E)$ is a soft closed set.

Conversely, let $c_\omega(F, E) - (F, E)$ be a soft closed set. Since (F, E) is soft $g\omega$ -closed, then by Theorem 2.9, $c_\omega(F, E) - (F, E)$ does not contain any non-empty soft closed set. Since $c_\omega(F, E) - (F, E)$ is a soft closed subset of itself, $c_\omega(F, E) - (F, E) = \emptyset$ and hence $c_\omega(F, E) = (F, E)$. $\square$

Theorem 2.12. Let ω be a soft weak structure on a soft topological space (X, τ, E) . Then (H, E) is soft $g\omega$ -open if and only if $(F, E) \tilde{\subset} i_\omega(H, E)$ whenever $(F, E) \tilde{\subset} (H, E)$ and (F, E) is soft closed.

Proof. Let (H, E) be a soft $g\omega$ -open set and $(F, E) \tilde{\subset} (H, E)$, where (F, E) is soft closed. Then $(H, E)'$ is a soft $g\omega$ -closed set contained in a soft open set $(F, E)'$. Hence $c_\omega(H, E)' \tilde{\subset} (F, E)'$, i.e. $(i_\omega(H, E))' \tilde{\subset} (F, E)'$. So $(F, E) \tilde{\subset} i_\omega(H, E)$. Conversely, Suppose that $(F, E) \tilde{\subset} i_\omega(H, E)$ for any soft closed set (F, E) whenever $(F, E) \tilde{\subset} (H, E)$. Let $(H, E)' \tilde{\subset} (G, E)$, where (G, E) is a soft open set. Then $(G, E)' \tilde{\subset} (H, E)$ and $(G, E)'$ is soft closed. By assumption, $(G, E)' \tilde{\subset} i_\omega(H, E)$ and hence $c_\omega(H, E) = (i_\omega(H, E))' \tilde{\subset} (G, E)$. Therefore, $(H, E)'$ is soft $g\omega$ -closed and hence (H, E) is soft $g\omega$ -open. $\square$

Theorem 2.13. Let ω be a soft weak structure on a soft topological space (X, τ, E) . Then the following are equivalent:

1. For every soft open (G, E) of X , $c_\omega(G, E) \tilde{\subset} (G, E)$.
2. Every soft subset of X is soft $g\omega$ -closed.

Proof. (1) $\implies$ (2) Let (F, E) be any soft subset of X where $(F, E) \tilde{\subset} (G, E)$ and (G, E) is any soft open set. Then by (1), $c_\omega(G, E) \tilde{\subset} (G, E)$ and hence $c_\omega(F, E) \tilde{\subset} c_\omega(G, E) \tilde{\subset} (G, E)$.

Thus (F, E) is soft $g\omega$ -closed.

(2) $\implies$ (1) Let (G, E) be any soft open set. Then by (2), (G, E) is soft $g\omega$ -closed and hence $c_\omega(G, E) \tilde{\subset} (G, E)$. $\square$

Theorem 2.14. Let ω be a soft weak structure on a soft topological space (X, τ, E) . If a soft set (H, E) is soft $g\omega$ -open and

if for a soft open set (G, E) , $i_\omega(H, E) \tilde{\cup} (H, E)' \tilde{\subset} (G, E)$, then $(G, E)' = \tilde{\emptyset}$.

Proof. Let (G, E) be a soft open set and $i_\omega(H, E) \tilde{\cup} (H, E)' \tilde{\subset} (G, E)$ for a soft $g\omega$ -open set (H, E) . Then $(G, E)' \tilde{\subset} (i_\omega(H, E))' \tilde{\cap} (H, E)$. This implies $(G, E)' \tilde{\subset} c_\omega(H, E) - (H, E)$. Since $(H, E)'$ is soft $g\omega$ -closed, by Theorem 2.9, $(G, E)' = \tilde{\emptyset}$. $\square$

Theorem 2.15. Let ω be a soft weak structure on a soft topological space (X, τ, E) . If a soft set (H, E) is soft $g\omega$ -open and $i_\omega(H, E) \tilde{\subset} (K, E) \tilde{\subset} (H, E)$, then (K, E) is soft $g\omega$ -open.

Proof. We have $(H, E)' \tilde{\subset} (K, E)' \tilde{\subset} c_\omega(H, E)'$. Since (H, E) is soft $g\omega$ -closed, from Theorem 2.7, it follows that $(K, E)'$ is soft $g\omega$ -closed, and hence (K, E) is soft $g\omega$ -open. $\square$

3. Separation Axioms

In this section, we introduce the new separation axiom, namely soft ω - $T_{\frac{1}{2}}$ -space in soft topological space with a soft weak structure ω .

Definition 3.1. A soft topological space (X, τ, E) with a soft weak structure ω is called a soft ω - $T_{\frac{1}{2}}$ -space if for every soft $g\omega$ -closed set (F, E) of X , $c_\omega(F, E) = (F, E)$

Theorem 3.2. Let ω be a soft weak structure on X . A soft topological space (X, τ, E) is soft ω - T_1 if (x, E) is soft ω -closed set for all $x \in X$.

Proof. Let $x, y \in X$ such that $x \neq y$. Then $(x, E)'$ and $(y, E)'$ are soft ω -open sets such $y \in (x, E)'$, $x \notin (x, E)'$ and $y \notin (y, E)'$, $x \in (y, E)'$. Hence X is soft ω - T_1 . $\square$

Corollary 3.3. If the union of soft ω -open sets is soft ω -open set, then the converse of Theorem 3.2, is true.

Theorem 3.4. Let ω be a soft weak structure on a soft topological space (X, τ, E) with respect to E . Then the following statements are equivalent:

1. X is a soft ω - $T_{\frac{1}{2}}$ -space.
2. Every singleton is either soft closed or $(x, E) = i_\omega(x, E)$

Proof. (1) $\implies$ (2). Suppose (x, E) is not a soft closed subset for some $x \in X$. Then $(x, E)'$ is not soft open, so if there is, then X is the only soft open set containing $(x, E)'$. Therefore $(x, E)'$ is soft $g\omega$ -closed. Since X is soft ω - $T_{\frac{1}{2}}$ -space, $c_\omega(x, E)' = (i_\omega(x, E))' = (x, E)'$ and thus $(x, E) = i_\omega(x, E)$.

(2) $\implies$ (1) Let (F, E) be a soft $g\omega$ -closed subset of X and $x \in c_\omega(F, E)$. We show that $x \in (F, E)$. If (x, E) is soft closed and $x \notin (F, E)$, then $x \in (c_\omega(F, E) - (F, E))$. Then $(x, E) \tilde{\subset} (F, E)'$ and hence $(F, E) \tilde{\subset} (x, E)'$. Since (F, E) is a soft $g\omega$ -closed set and $(x, E)'$ is a soft open, $c_\omega(F, E) \tilde{\subset} (x, E)'$ and hence $(x, E) \tilde{\subset} (c_\omega(F, E))'$. Therefore, $(x, E) \tilde{\subset} c_\omega(F, E) \tilde{\cap} (c_\omega(F, E))' = \emptyset$. This is a contradiction. Therefore, $x \in (F, E)$. If $(x, E) = i_\omega(x, E)$, since $x \in c_\omega(F, E)$, then for every soft ω -open set (G, E) such that $x \in (G, E)$, we have $(G, E) \tilde{\cap} (F, E) \neq \emptyset$. By assumption $(x, E) = i_\omega(x, E)$, we have (x, E) be an ω -open soft set and $(x, E) \tilde{\cap} (F, E) \neq \emptyset$. Hence $x \in (F, E)$. Therefore, in both cases we have $x \in (F, E)$. Therefore, $c_\omega(F, E) = (F, E)$ and hence X is a soft ω - $T_{\frac{1}{2}}$ -space. $\square$

Theorem 3.5. If the union of soft ω -open sets is soft ω -open set in a soft weak structure ω on a soft topological space, then every soft ω - T_1 -space is a soft ω - $T_{\frac{1}{2}}$ -space.

Proof. Suppose that (X, τ, E) is a soft T_1 -space and the union of soft ω -open sets is soft ω -open set. It suffices to show that a set which is not soft ω -closed also is not soft $g\omega$ -closed set. Let (F, E) is not soft ω -closed. Let $x \in c_\omega(F, E) - (F, E)$. Then $(x, E) \tilde{\subset} c_\omega(F, E) - (F, E)$ and (x, E) is a nonempty soft ω -closed set in X by Corollary 3.3. Hence, by Theorem 2.9, (F, E) is not soft $g\omega$ -closed. $\square$

The next example shows that the converse of the above theorem is not true in general.

Example 3.6. Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\emptyset, \tilde{X}, (F_1, E), (F_2, E)\}$ where

$$\begin{aligned} F_1(e_1) &= \{h_1\}, & F_1(e_2) &= \{h_1\}; \\ F_2(e_1) &= \{h_1, h_2\}, & F_2(e_2) &= X; \end{aligned}$$

Let $\omega = \{\emptyset, (F_1, E), (F_2, E)\}$ be an ω over X with respect to E . Then (X, τ, E) is soft ω - $T_{\frac{1}{2}}$ -space but not soft ω - T_1 .

Theorem 3.7. Let ω be a soft weak structure on a soft topological space (X, τ, E) . If X is soft ω - T_0 , then for each $x, y \in X$ such that $x \neq y$, we have $c_\omega(x, E) \not\tilde{\subset} c_\omega(y, E)$.

Proof. Let X be a soft ω - T_0 and $x, y \in X$ such that $x \not\tilde{=}_\omega y$. Then there exists soft ω -open set (F, E) such that $x \in (F, E)$ and $y \notin (F, E)$. Therefore $(F, E)'$ is soft ω -closed set such that $x \notin (F, E)'$ and $y \in (F, E)'$. Since $c_\omega(y, E)$ is the intersection of all soft ω -closed subsets that contain y , then $c_\omega(y, E) \tilde{\subset} (F, E)'$ and hence $x \notin c_\omega(y, E)$. Thus $c_\omega(x, E) \not\tilde{\subset} c_\omega(y, E)$. $\square$

Corollary 3.8. If $c_\omega(x, E)$ is a soft ω -closed set for each $x \in X$, and if for each distinct $x, y \in X$, $c_\omega(x, E) \not\tilde{\subset} c_\omega(y, E)$, then X is soft ω - T_0 .

Proof. For each distinct $x, y \in X$, since $c_\omega(x, E) \not\tilde{\subset} c_\omega(y, E)$, there exists some $z \in X$ such that $z \in c_\omega(x, E)$ and $z \notin c_\omega(y, E)$. If $x \in c_\omega(y, E)$, then $c_\omega(x, E) \tilde{\subset} c_\omega(y, E)$ which is a contradiction since $z \notin c_\omega(y, E)$. Thus $(c_\omega(y, E))'$ is soft ω -open set such that $x \in (c_\omega(y, E))'$ and $y \notin (c_\omega(y, E))'$. Hence X is soft ω - T_0 . $\square$

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

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