Periodic Doubling and Chaotic Attractor in the Love Model with a Fourier Series Function as External Force

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Abstract

This paper propose a dynamic mathematical model of love with the Fourier series function as an external force. We also investigate the periodic doubling including 1, 2, 4 periodic motion, chaotic attractor and limit cycle in the love models based on Romeo and Juliet with the Fourier series function as an external force, using time series and phase portraits. To show the nonlinear phenomena using time series and phase portraits, we vary the parameter value with the Fourier series function as an external force.

Keywords: Chaotic phenomena, Nonlinear dynamic, Love model, Time series, Phase portrait, External force, Fourier series function

1. Introduction

In general, the human society can model as a complex system. Typically, complex systems have the characteristics of a nonlinear dynamical system, which can describe by a differential equation or difference equation. However, when we compare with linear systems, nonlinear systems are more difficult to analyze, synthesize, implement or design. All natural systems in the world or earth such as wind, weather, and human behavior are nonlinear systems, which include fuzzy systems, neural networks, chaotic systems, and complex systems. Among these nonlinear systems, chaotic systems have gained considerable interest over the last three decades among many researches in the field of mathematics, physics, chemistry, engineering, and social science.

In particular, studies on chaotic behaviors in the social sciences, including those related to habits and the human mind, such as addiction [1–4], happiness [5–8], physical exercise [27] and love models [8–13], involve an overlap of the fields of mathematics, biology, psychology, and social science.

Bae and his associates [1–4] and Kim [27] proposed a mathematical model for addictions to digital leisure, internet, tobacco, and physical exercise. Further, they also verified the presence of nonlinear behaviors or chaotic phenomena by using time series and phase portraits.

Sprott [6, 8] proposed a basic dynamic equation for happiness and investigated its behavior. Bae [7] proposed a mathematical happiness model and a synchronization technique for the model.

Love model have proposed several models including the Romeo and Juliet model [9–13], the Laura and Petrarch model [14, 15], the Adam and Eve model [16], and others [17–21].

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Among these, the Romeo and Juliet's love model is most commonly used in the research of nonlinear dynamics and complex systems.

Strogatz [21, 22] was the first to suggest modelling love affairs by using differential equations; for example, a simple model for the ill-fated romance of Romeo and Juliet modelled as a second-linear system. He also defined love affairs according to parameter values.

Sprott [8] who is inspired by Strogatz's research [21, 22], proposed a love model based on Romeo and Juliet, which was represented by the linear differential equation; Sprott also described the linear as well as nonlinear behaviors of the model.

The existence of periodic motion and chaotic behavior or motion in the love model based on Romeo and Juliet is represented through time series and phase portraits, with either the same or different time delays, an external force, and different external forces proposed by Bae [16–21, 25, 26].

Bae and his associates recently proposed external force that compose fuzzy membership function such as Gaussian [28, 29], triangular [30], which close to represent human mind. They shows chaotic behavior according to varying parameter value.

However, most previously published papers in the love model of Romeo and Juliet, sufficiently have not provided to represent human mind through an time series or phase portrait to demonstrate chaotic phenomena or behavior, including periodic motion.

In this paper, we propose a dynamic mathematical model of love with the Fourier series function as an external force. We also investigate the chaotic behaviors including periodic doubling in the love models based on Romeo and Juliet with the Fourier series function as an external force based on varying the parameter value using time series and phase portraits.

2. Love Model

There are many love models for Romeo and Juliet; we can classify these models as follows:

2.1 Basic Love Model

Strogatz [21, 22] and Sprott [15] proposed the basic love model for Romeo and Juliet, which can formulate as

$$\begin{aligned}\frac{dR}{dt} &= aR + bJ, \\ \frac{dJ}{dt} &= cR + dJ,\end{aligned}\quad (1)$$

where parameter a , and b specify Romeo's romantic style, and c and d specify Juliet's style.

2.2 Alternative Love Model

Sprott [8] also proposed an alternative love model for Romeo and Juliet, which can be written as

$$\begin{aligned}\frac{dR}{dt} &= aR + bJ(1 - |J|), \\ \frac{dJ}{dt} &= cR(1 - |R|) + dJ,\end{aligned}\quad (2)$$

Where, parameter a , b , c and d are same to use for the basic love model.

2.3 Alternative Love Model with an External Force

(1) and (2) cannot shows the chaotic phenomena, because these equations have second-order systems. To make chaotic phenomena in a dynamic model, the model must involve a third-order system, and it must contain at least one nonlinear term. Because (1) and (2) for the love model of Romeo and Juliet do not satisfy the necessary conditions for a third-order system, they have to modify to satisfy the condition. Bae [16] proposed an alternative love model, which involves external forces, can be written as

$$\begin{aligned}\frac{dR}{dt} &= aR + bJ(1 - |J|) + f(t), \\ \frac{dJ}{dt} &= cR(1 - |R|) + dJ,\end{aligned}\quad (3)$$

or

$$\begin{aligned}\frac{dR}{dt} &= aR + bJ(1 - |J|), \\ \frac{dJ}{dt} &= cR(1 - |R|) + dJ + y(t),\end{aligned}\quad (4)$$

or

$$\begin{aligned}\frac{dR}{dt} &= aR + bJ(1 - |J|) + f(t), \\ \frac{dJ}{dt} &= cR(1 - |R|) + dJ + y(t),\end{aligned}\quad (5)$$

where $f(t)$ and $y(t)$ are external forces applied to the Romeo and Juliet differential equations, respectively.

Authors [24, 25] demonstrated the existence of chaotic phenomena when $f(t) = \sin \omega t$ and $y(t) = 0$ in (3), $f(t) = 0$ and $y(t) = \sin \omega t$ in (4), and $f(t) = \sin \omega t$ and $y(t) = \sin \omega t$ in (5), using time series and phase portraits, which include

Figure 1. Fourier series function.

periodic motions and chaotic attractors.

3. Chaotic Behaviors in Love Model with Fourier Series Function as an External Force

In this paper, we consider the chaotic behaviors in (3), (4), and (5) with the Fourier series as an external force using computer simulation with MATLAB. In addition, we apply different parameters for (3), (4), and (5). Equation (6) represents the Fourier series function that use as an external force, and Figure 1 shows its time series.

$$f(t) = 5 \left(\sin \pi t + \frac{1}{3} \sin 3\pi t + \frac{1}{5} \sin 5\pi t + \frac{1}{7} \sin 7\pi t \dots \right) \quad (6)$$

In the following section, the time series and phase portrait for (4) are reviewed with different parameter values; in different cases when external force is applied as a Fourier series function. The parameter b , c , and d are fixed as -2 , 1 , and 1 , respectively. When parameter a varied from -7 to 12 , time series and phase portrait get following as.

3.1 Case from $a = -7$ to $a = -5.6$

First, we investigate the time series and phase portrait to show the behavior of love for Romeo and Juliet, when we set the parameters “ a ” from -7 to -5.6 in (4), which has the characteristics of an alternative love model. Figure 2 illustrates the results of the time series and phase portrait, when the parameter “ a ” have -7 in (4) that are applied for Juliet’s item with Fourier

(a)

(b)

Figure 2. Results of time series (a) and phase portrait (b) when (4) apply with $a = -6.5$, $b = -2$, $c = 1$, and $d = 1$ parameters.

series function as an external forces. From Figure 2, we recognize that this is 1-periodic motion. 1-periodic motion maintain until $a = -5.6$.

3.2 Case $a = -5.5$

We investigate the time series and phase portrait to show the behavior of love for Romeo and Juliet when we set the parameters as $a = -5.5$, $b = -2$, $c = 1$, and $d = 1$ in (4). Figure 3 shows the results of the time series and phase portrait. We know that Figure 3 represents 2-periodic motion and its motion continue

(a)

(b)

Figure 3. Results of time series (a) and phase portrait (b) when (2) apply with $a = -5.5$, $b = -2$, $c = 1$, and $d = 1$ parameters.

to until $a = -5.1$.

3.3 Case $a = -5$

We now investigate the time series and phase portrait to show the behavior of love for Romeo and Juliet when we set the parameters as $a = -5$, $b = -2$, $c = 1$, and $d = 1$ in (4). Figure 4 illustrates the results of the time series and phase portrait with these parameters. From Figure 4, we know it has 4-periodic motion. We also see from Figures 2-4 that these figures progress

(a)

(b)

Figure 4. Results of time series (a) and phase portrait (b) when (2) apply with $a = -5.0$, $b = -2$, $c = 1$, and $d = 1$ parameters.

periodic motion of 1, 2, 4, which is called periodic doubling that take place typically before chaotic motion. Therefore, we can expect that chaotic attractor appear next parameter value.

3.4 Case $a = -4.5$

We now investigate the chaotic attractor to discover the nature of the chaotic phenomena, when we set the parameters as $a = -4.5$, $b = -2$, $c = 1$, and $d = 1$, with the Fourier series function in (4). Figure 5 illustrates the results of the time series and phase portrait with these parameters. On comparing Figures

Figure 5. Results of time series (a) and phase portrait (b) when (2) apply with $a = -4.5$, $b = -2$, $c = 1$, and $d = 1$ parameters.

4 and 5, we recognize that there is a significant difference, one is periodic motion and the other is chaotic motion as varying different parameters.

Generally chaotic phenomena happens following as order: periodic motion, chaotic motion, periodic motion which is called periodic windows and limit cycle, or periodic motion, chaotic motion and limit cycle. Thus, we can also expect that two different way, periodic motion and chaotic attractor, and limit cycle occur when we apply another parameter value.

3.5 Case $a = -4$ and $a = -3.5$

We investigate that periodic motion or limit cycle take place when we set the parameters as $a = -4$, $b = -2$, $c = 1$, and $d = 1$, with the Fourier series function in (4). Figures 6 and 7 illustrates the results of the time series and phase portrait with these parameters. From Figures 6 and 7, we know that these figures seems like periodic motion. In this case, the periodic motion cannot shows periodic doubling, they shows various periodic motion such as 3-periodic motion or quasi-periodic motion. Figure 6 shows simple periodic motion whereas Figure 7 shows quasi-periodic motion.

From result of Figures 5 and 6, we can easily expect that next parameter value shows the chaotic motion because of pattern according to order of periodic doubling, chaotic attractor, periodic window and limit cycle.

3.6 Case $a = -3.0$

We investigate that periodic motion or limit cycle take place when we set the parameters as $a = -3.0$, $b = -2$, $c = 1$, and $d = 1$, with the Fourier series function in (4).

From result of Figures 5 and 7, we recognize that the 2 chaotic attractors are little different.

3.7 Case from $a = -2.5$ to $a = -1$

We investigate the limit cycle when we set the parameters as $a = -2.5$, $b = -2$, $c = 1$, and $d = 1$, with the Fourier series function in (4). Figure 8 shows the results of the time series and phase portrait with these parameters.

4. Conclusions

In this paper, we proposed a dynamic mathematical model of love with a Fourier series function as an external force. We also investigated the chaotic behaviors that have periodic doubling and chaotic attractor in love models of Romeo and Juliet with the Fourier series function as an external force, using time series and phase portraits. This paper shows that proposed dynamic mathematical model of love with a Fourier series function as an external force have chaotic behaviors including periodic doubling, chaotic attractor and limit cycle.

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

(a)

(b)

Figure 6. Results of time series (a) and phase portrait (b) when (2) apply with $a = -4.0$, $b = -2$, $c = 1$, and $d = 1$ parameters.

(a)

(b)

Figure 7. Results of time series (a) and phase portrait (b) when (2) apply with $a = -3.5$, $b = -2$, $c = 1$, and $d = 1$ parameters.

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(a)

(b)

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(a)

(b)

Figure 9. Results of time series (a) and phase portrait (b) when (2) apply with $a = -2.5$, $b = -2$, $c = 1$, and $d = 1$ parameters.

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