Direct Adaptive Fuzzy Sliding Mode Control for Under-actuated Uncertain Systems

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Abstract

The development of the control algorithms for under-actuated systems is important. Decoupled sliding mode control has been successfully employed to control under-actuated systems in a decoupling manner with the use of sliding mode control. However, in such a control scheme, the system functions must be known. If there are uncertainties in those functions, the control performance may not be satisfactory. In this paper, the direct adaptive fuzzy sliding mode control is employed to control a class of under-actuated uncertain systems which can be regarded as a combination of several subsystems with one same control input. By using the hierarchical sliding control approach, a sliding control law is derived so as to make every subsystem stabilized at the same time. But, since the system considered is assumed to be uncertain, the sliding control law cannot be readily facilitated. Therefore, in the study, based on Lyapunov stable theory a fuzzy compensator is proposed to approximate the uncertain part of the sliding control law. From those simulations, it can be concluded that the proposed compensator can indeed cope with system uncertainties. Besides, it can be found that the proposed compensator also provide good robustness properties.

Keywords: Adaptive fuzzy control, Under-actuated systems, Sliding mode control

1. Introduction

Most mechanical systems, such as individual links of robotic manipulators, are assumed to be directly controllable; that is, all components of those systems are wished to be individually actuated. However, it is not always possible because there may exist failure or attrition in those components and actuators. In such cases, the system becomes under-actuated. An under-actuated system is a system having a fewer number of control inputs than the degree of freedom of the system to be controlled [1]. In fact, there also exist many under-actuated systems in real applications such as mentioned in [1, 2], free-flying space robots, underwater robots, walking robots, mobile robots, flexible-link robots, surface vessels, helicopters, etc. The investigations of under-actuated mechanical systems are valuable in many applications. For instance, if under-actuated control can work well, the number of actuators can be reduced so that the weight or size of the system can become more economical. Such advantages of considering under-actuated mechanical systems can also be found for walking robots, aircrafts,

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spacecrafts, etc. Sometimes, the control algorithms for under-actuated systems can be utilized to partially retrieve the functions of a malfunction-plant. By a proper under-actuated control algorithm, as presented in [3, 4], the robot arm still can provide partial functions. Therefore, the development of the control algorithms for under-actuated systems is important.

In [2], a way of transforming an under-actuated system in a Lagrange equation form into a decoupled representation is proposed. Then, the control methods about decoupled systems have been discussed for years and can be used to design controllers for under-actuated systems. The idea is to partition the system into several subsystems, and each subsystem can be treated as a second order canonical form system so that the control of the whole system can easily be designed. Sliding mode control is widely used to cope with many nonlinear systems. As mentioned in [5-12], major advantages of sliding mode control include insensitive to external disturbances, robustness against system uncertainty, stability guaranteed of the system, fast dynamic response, and simple to implement. In [13], a multi-level (hierarchical) sliding model based decoupling control was proposed to successfully stabilize a class of under-actuated systems. In the approach, a sliding surface is defined for each subsystem respectively and they are linearly combined to form a so-called decoupling sliding surface. Then, the control law can be derived based on the decoupling sliding surface. With different decoupling sliding surface designs, the obtained control law is adaptive as presented in [14, 15]. However, there are several parameters required in the decoupling sliding surface. Since these parameters are not easily identified, several estimators for those parameters have been developed in [10, 11, 16]. Instead of estimating those parameters, in this study, we intent to directly employ adaptive fuzzy control schemes for the design of the decoupling sliding surface [13]. From our simulation, it can be found that such an approach is indeed effective.

Fuzzy control has been widely used in many applications [8, 10]. Based on the property that a fuzzy system can be an universal function approximator, as proved in [17, 18], the so-called adaptive fuzzy controls have been developed by using the feedback linearization or the backstepping approach as discussed in [19]. This kind of approaches is an important methodology for dealing with system uncertainties. In those approaches, the consequent of a considered fuzzy system (approximator) is updated by a specified adaptive law, which is often derived based on the Lyapunov stable theory. If such a fuzzy system is utilized to directly approximate the designated reference control law, it is often referred to as the direct adaptive fuzzy control [20]. In

this paper, the direct adaptive fuzzy sliding mode control will be considered in defining the decoupling sliding surface for a class of under-actuated systems.

As mentioned, a direct adaptive fuzzy sliding mode control system is proposed based on the control design discussed in [13]. According to the proposed decoupling sliding surface in [13] and the feedback linearization control technique, a sliding control law is proposed in our study. This control law consists of an equivalent control law and a hitting control law. However, since the under-actuated system considered in this study is assumed to be uncertain, the equivalent control part of the sliding control law is not available because the system functions are unknown. Thus, the direct adaptive fuzzy control is employed to approximate the uncertain equivalent control part, where the input of the fuzzy approximator is defined as the decoupling sliding surface. The detailed proposed control scheme will be introduced in the following sections.

The remainder of this paper is organized as follows: in Section 2, the considered under-actuated system is described and the model based sliding control law is derived. In Section 3, the design of the fuzzy approximator is briefly introduced. In Section 4, the proposed direct adaptive fuzzy sliding mode control system is presented. In Section 5, the commonly-used double-inverted pendulum system is considered as the test platform to demonstrate the effectiveness of the proposed designs. Finally, the conclusions are given in Section 6.

2. System Description

In this study, a single-input-multiple-output under-actuated system is considered and can be described as:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = f_1(x) + b_1(x)u(t) \\ \dot{x}_3(t) = x_4(t) \\ \dot{x}_4(t) = f_2(x) + b_2(x)u(t) \\ \vdots \\ \dot{x}_{n-1}(t) = x_n(t) \\ \dot{x}_n(t) = f_m(x) + b_m(x)u(t) \end{cases} \quad (1)$$

where $f_1(x)$, $f_2(x)$, $\dots$, and $f_m(x)$ are system functions with uncertainties, $b_1(x)$, $b_2(x)$, $\dots$ and $b_m(x)$ are system gain functions and are supposed to be known, $u(t)$ is the control input of the system, m is the number of the sub-systems, each of which is a second order system, and n is the number of states.

Then, it is easy to verify that $n = 2m$. The state vector can be written by $x = [x_1, x_2, \dots, x_n]^T$. The system can be regarded as m sub-systems $(\varphi_1, \varphi_2, \dots, \varphi_m)$ with second-order canonical forms, as

$$\varphi : \begin{cases} \varphi_1 : \begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = f_1(x) + b_1(x)u \end{cases} \\ \varphi_2 : \begin{cases} \dot{x}_3(t) = x_4(t) \\ \dot{x}_4(t) = f_2(x) + b_2(x)u \end{cases} \\ \vdots \\ \varphi_m : \begin{cases} \dot{x}_{n-1}(t) = x_n(t) \\ \dot{x}_n(t) = f_m(x) + b_m(x)u \end{cases} \end{cases} \quad (2)$$

As mentioned, those system functions are subject to uncertainties. Thus, we assume that they can be written as

$$\begin{cases} f_1(x) = \hat{f}_1(x) + \Delta f_1(x) \\ f_2(x) = \hat{f}_2(x) + \Delta f_2(x) \\ \vdots \\ f_m(x) = \hat{f}_m(x) + \Delta f_m(x) \end{cases} \quad (3)$$

where $\hat{f}_i(x) |_{i=1,2,\dots,m}$ represents the nominal function of $f_i(x)$ and $\Delta f_i(x) |_{i=1,2,\dots,m}$ represents the uncertain term of $f_i(x)$. Hence, system (1) is rewritten as

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = \hat{f}_1(x) + \Delta f_1(x) + b_1(x)u(t) \\ \dot{x}_3(t) = x_4(t) \\ \dot{x}_4(t) = \hat{f}_2(x) + \Delta f_2(x) + b_2(x)u(t) \\ \vdots \\ \dot{x}_{n-1}(t) = x_n(t) \\ \dot{x}_n(t) = \hat{f}_m(x) + \Delta f_m(x) + b_m(x)u(t) \end{cases} \quad (4)$$

As mentioned, the sliding mode control is considered in this study. Thus, the system is divided into m subsystems that are in canonical forms and each subsystem is controlled by its corresponding sliding surface. The sliding surfaces of those subsystems are defined as:

$$\begin{cases} s_1 = c_1 x_1 + x_2 \\ s_2 = c_2 x_3 + x_4 \\ \vdots \\ s_m = c_m x_{n-1} + x_n \end{cases} \quad (5)$$

where $c_i |_{i=1,2,\dots,m} > 0$ are bounded sliding coefficients and usually are selected in a trial and error manner. The basic idea is to design a controller based on those sliding surface so that the system can have required control performance.

Now, consider the first system, (φ_1) . Choose a Lyapunov function as

$$V_1 = \frac{1}{2} s_1^2 \quad (6)$$

It is desired that the controller can make $\dot{V}_1$ negative to guarantee the first subsystem stable. In other words, we hope

$$\dot{V}_1 = s_1 \dot{s}_1 = s_1 (c_1 x_2 + f_1 + b_1 u) < 0 \quad (7)$$

In the sliding mode control design, the controller can be selected as

$$u = \frac{1}{b_1} [-f_1 - c_1 x_2 - k_1 \text{sgn}(s_1)] = u_{\text{eq}1} + u_{\text{hit}1} \quad (8)$$

where k_1 is a positive bounded constant. Here,

$$u_{\text{eq}1} = \frac{1}{b_1} [-f_1 - c_1 x_2]$$

is referred to as the equivalent control input and

$$u_{\text{hit}1} = \frac{1}{b_1} [-k_1 \text{sgn}(s_1)]$$

is the hitting control input. With the same idea, the control inputs of the other subsystems can also be obtained.

In above, all subsystems are ensured to be stable by their individual controllers. However, since the system considered is under-actuated and there is only one control input. Thus, a way of combining all controllers into one must be proposed. In [13], a hierarchical sliding surface is proposed and is written as

$$s_c = s_1 - k_{d2} s_2 + k_{d3} s_3 - \dots \pm k_{dm} s_m \quad (9)$$

where k_{d2} , k_{d3} , ... and k_{dm} are positive constants and are called decoupling constants, and $\pm$ is a sign symbol for the m -th term. When m is odd, $\pm$ is $+$; otherwise, $\pm$ is $-$. Decoupling constants are determined in a trial and error manner.

Similarly, a Lyapunov function is defined as

$$V = \frac{1}{2} s_c^2. \quad (10)$$

Then, we hope to have

$$\begin{aligned}
\dot{V} &= s_c \dot{s}_c \\
&= s_c \{c_1 x_2 + f_1 - k_{d2} c_2 x_4 - k_{d2} f_2 \\
&\quad \cdots \pm k_{dm} c_m x_n \pm k_{dm} f_m \\
&\quad + (b_1 - k_{d2} b_2 + k_{d3} b_3 \cdots \pm k_{dm} b_m) u\} \\
&< 0
\end{aligned} \tag{11}$$

Thus, the controller can be selected as

$$\begin{aligned}
u^* &= \frac{1}{b_1 - k_{d2} b_2 + k_{d3} b_3 \cdots \pm k_{dm} b_m} \\
&\quad \times \{-c_1 x_2 - f_1 + k_{d2} c_2 x_4 + k_{d2} f_2 \cdots \\
&\quad \mp k_{dm} c_m x_n \mp k_{dm} f_m - k_c \operatorname{sgn}(s_c)\} \\
&= \frac{1}{b_1 - k_{d2} b_2 + k_{d3} b_3 \cdots \pm k_{dm} b_m} \\
&\quad \times \{-c_1 x_2 - f_1 + k_{d2} c_2 x_4 + k_{d2} f_2 \\
&\quad \cdots \mp k_{dm} c_m x_n \mp k_{dm} f_m\} \\
&\quad + \frac{\{-k_c \operatorname{sgn}(s_c)\}}{b_1 - k_{d2} b_2 + k_{d3} b_3 \cdots \pm k_{dm} b_m} \\
&= u_{eq} + u_{hit}
\end{aligned} \tag{12}$$

where k_c is a positive bounded constant. Again, the equivalent control (u_{eq}) and the hitting control (u_{hit}) can be defined accordingly. By substituting Eq. (12) into Eq. (11), we have $\dot{V} < 0 |_{s_c \neq 0}$ and $\dot{V} = 0 |_{s_c = 0}$. Hence, system (1) is asymptotically stable.

However, it should be noted that the control input u^* need to have knowledge about the system functions, $f_i(x) |_{i=1,2,\dots,m}$. But, as mentioned earlier, those system functions are subject to uncertainties or even unknown. In the literature, there is an approach called adaptive fuzzy control that has been proposed to resolve this kind of problems [13, 20-24]. The idea of direct adaptive fuzzy control is to employ a fuzzy approximator with a finite number of fuzzy rules to online approximate (estimate) the desired controller. In our study, we also employ this kind of approach to approximate the equivalent control. This approach is called the decoupling sliding direct adaptive fuzzy control.

3. Fuzzy Approximator

As mentioned, the direct adaptive fuzzy control is to use a fuzzy approximator (system) to approximate an uncertain control law directly. In the process, an adaptive law must be obtained to update the fuzzy approximator. In this section, the fuzzy approximator is briefly introduced.

The fuzzy approximator used consists of a set of fuzzy IF-

THEN rules as:

$$\begin{aligned}
&\text{Rule-}l: \text{ IF } s_c \text{ is } A_l \\
&\text{ THEN } y_F \text{ is } \theta_d^l \text{ for } l = 1, 2, 3, \dots, M
\end{aligned} \tag{13}$$

where s_c is the input, y_F is the output, A_l is the l -th rule, θ_d^l is the fuzzy singleton corresponding to fuzzy sets, and M is the number of fuzzy rules. Then, the overall output of the fuzzy approximator is

$$y_f(s_c | \theta_d^l) = \frac{\sum_{l=1}^M \theta_d^l \mu_{A_l}}{\sum_{l=1}^M \mu_{A_l}}, \tag{14}$$

where $\mu_{A_l}(s_c)$ is the membership function of fuzzy set A_l . Eq. (14) can be written in a linear-in-parameter (LIP) form as:

$$y_f(s_c | \theta_d^l) = \theta_d^T \omega_d \tag{15}$$

where $\theta_d^T = [\theta_d^1, \theta_d^2, \theta_d^3, \dots, \theta_d^M]$ and is called the adjustable parameter vector, and $\omega_d = [\omega_d^1, \omega_d^2, \omega_d^3, \dots, \omega_d^M]^T$ and is a regressive vector, where

$$\omega_d^l = \frac{\mu_{A_l}}{\sum_{l=1}^M \mu_{A_l}}, l = 1, 2, 3, \dots, M \tag{16}$$

As mentioned, this approximator is directly employed to model the control input.

Since the number of fuzzy rules is finite, approximation errors must be taken into consideration as in [19]. A robust control law denoted as u_r is considered into the control law to cope with possible approximator error and/or external disturbance, and then the control input becomes

$$u = u_d + u_r = \theta_d^T \omega_d + u_r. \tag{17}$$

In fact, in Eq. (12), there is a hitting control term, which can also be viewed as a robust control. Thus, in our approach, the fuzzy approximator will be employed to estimate u_{eq} in Eq. (12) and the hitting control u_{hit} will be considered as u_r .

By the knowledge in [19], Eq. (14) is a universal approximator. In [13], the optimal fuzzy approximator can be denoted as

$$u_d^*(s_c | \theta_d^*) = u_d^* = \theta_d^{*T} \omega_d \tag{18}$$

where $\theta_d^{*T} = [\theta_d^{*1}, \theta_d^{*2}, \dots, \theta_d^{*M}]$ is an optimal parameter vector and is defined as:

$$\theta_d^* \equiv \arg \min_{\theta_d \in \Omega_{\theta_d}} \left\{ \sup_{s_c \in \Omega_{s_c}} |u^* - u_d^*(s_c | \theta_d)| \right\} \tag{19}$$

where Ω_{θ_d} and Ω_s are the constant sets of θ_d and s_c , respectively. In the following, the adaptive control law of the fuzzy approximator will be derived.

4. Adaptive Fuzzy Sliding Mode Control

In the section, the adaptive law for the fuzzy approximator as (15) that can online approximate the uncertain equivalent control law in (12) is proposed. The whole system stability is also shown in the section.

By adding and subtracting u^* into Eq. (10), we have

$$\begin{aligned}\dot{s}_c &= c_1 x_2 + f_1 - k_{d2} c_2 x_4 - k_{d2} f_2 \cdots \\ &\quad \pm k_{dm} c_m x_n \pm k_{dm} f_m + \\ &\quad (b_1 - k_{d2} b_2 + k_{d3} b_3 \cdots \pm k_{dm} b_m) (u + u^* - u^*) \\ &= (b_1 - k_{d2} b_2 + k_{d3} b_3 \cdots \pm k_{dm} b_m) (u - u^*) - k_c \operatorname{sgn}(s_c)\end{aligned}\quad (20)$$

Then, let $u = u_d$ and we have

$$\begin{aligned}\dot{s}_c &= - (b_1 - k_{d2} b_2 + k_{d3} b_3 \cdots \pm k_{dm} b_m) \\ &\quad (u^* - u_d^* + u_d^* - u_d) - k_c \operatorname{sgn}(s_c) \\ &= - (b_1 - k_{d2} b_2 + k_{d3} b_3 \cdots \pm k_{dm} b_m) \\ &\quad (\varepsilon + \tilde{\theta}_d^T \omega_d) - k_c \operatorname{sgn}(s_c)\end{aligned}\quad (21)$$

where $\varepsilon_d = u^* - u_d^*$ is the approximation error and $\tilde{\theta}_d^T \omega_d = u_d^* - u_d$ is the learning error with $\tilde{\theta}_d^T = (\theta_d^* - \theta_d)^T$. With the property of universal approximation theory for fuzzy approximators as stated in [18], $\varepsilon_d \in L_\infty$ is valid.

Now, define a Lyapunov function as:

$$V = \frac{1}{2} s_c^2 + \frac{1}{2\alpha} \tilde{\theta}_d^T \tilde{\theta}_d \quad (22)$$

where $\alpha > 0$ is the learning rate of the fuzzy compensator. To guarantee the system stable, it is desired that $\dot{V}$ must less than 0. It can be obtained that

$$\begin{aligned}\dot{V} &= s_c \dot{s}_c - \frac{1}{\alpha} \tilde{\theta}_d^T \dot{\theta}_d \\ &= s_c (b_1 - k_{d2} b_2 + k_{d3} b_3 \cdots \pm k_{dm} b_m) (\varepsilon_d + \tilde{\theta}_d^T \omega_d) \\ &\quad p - k_c |s_c| - \frac{1}{\alpha} \tilde{\theta}_d^T \dot{\theta}_d\end{aligned}\quad (23)$$

If the adaptive law is

$$\dot{\theta}_d = \alpha s_c (b_1 - k_{d2} b_2 + k_{d3} b_3 \cdots \pm k_{dm} b_m) \omega_d, \quad (24)$$

We can have

$$\dot{V} = -s_c (b_1 - k_{d2} b_2 + k_{d3} b_3 \cdots \pm k_{dm} b_m) (\varepsilon_d - u_r) - k_c |s_c| \quad (25)$$

Since $\varepsilon_d \in L_\infty$, if a suitable k_c can be selected to suppress the approximation error, $\dot{V} < 0|_{s_c \neq 0}$ and $\dot{V} = 0|_{s_c = 0}$ can be achieved. In fact, similar to traditional robust control schemes, k_c can be selected to be a bound of the related function as discussed in [25]. Therefore, it is evident that the system can be stabilized by the adaptive fuzzy control law as (25).

5. Simulations

In our study, in order to demonstrate the effects of the proposed approach, a physical system, a doubled-inverted pendulum system is considered as the platform for control. The doubled-inverted pendulum system is often used for verifying theoretical conjecture such as in [9-11, 13]. It is a 6-order system and an under-actuated SIMO system with three degrees of freedom. The mathematical model and parameters of the considered doubled-inverted pendulum system are referred to [13]. The system is shown in Figure 1 and is described in a decoupled form as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1 + b_1 u \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2 + b_2 u \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = f_3 + b_3 u \end{cases} \quad (26)$$

where

$$\begin{aligned}f_1 &= \frac{A_{21}}{l_1 m_1} \sin(\theta_2 - \theta_1) + \frac{1}{l_1} g \sin \theta_1 - \frac{A_{11}}{l_1 m_c} \cos \theta_1 \sin \theta_1, \\ b_1 &= \frac{A_{22}}{l_1 m_1} \sin(\theta_2 - \theta_1) - \frac{1}{l_1 m_c} \cos \theta_1 - \frac{A_{12}}{l_1 m_c} \cos \theta_1 \sin \theta_1,\end{aligned}$$

$$f_2 = \frac{A_{11}}{l_2 m_1} \sin(\theta_2 - \theta_1),$$

$$b_2 = \frac{A_{12}}{l_2 m_1} \sin(\theta_2 - \theta_1),$$

$$f_3 = \frac{A_{11}}{m_c} \sin \theta_1,$$

and

$$b_3 = \frac{1}{m_c} + \frac{A_{12}}{m_c} \sin \theta_1$$

with

$$\begin{aligned}
A_{11} &= \frac{a_{22}(l_1\dot{\theta}_1^2 - g \cos \theta_1) - a_{12}l_2\dot{\theta}_2^2}{\Delta}, \\
A_{12} &= -\frac{a_{22} \sin \theta_1}{\Delta \cdot m_c}, \\
A_{21} &= \frac{-a_{12}(l_1\dot{\theta}_1^2 - g \cos \theta_1) + a_{11}l_2\dot{\theta}_2^2}{\Delta}, \\
A_{22} &= \frac{a_{12} \sin \theta_1}{\Delta \cdot m_c}, \\
a_{11} &= \frac{1}{m_1} + \frac{1}{m_c} \sin^2 \theta_1, \\
a_{12} &= -\frac{\cos(\theta_2 - \theta_1)}{m_1}, \\
a_{22} &= \frac{1}{m_1} + \frac{1}{m_2},
\end{aligned}$$

and

$$\Delta = a_{11}a_{22} - a_{12}^2.$$

Here, $x_1 = \theta_1$ is the angle of pole 1 with respect to the vertical axis, $x_2 = \dot{\theta}_1$ is the angle velocity of pole 1 with respect to the vertical axis, $x_3 = \theta_2$ is the angle of pole 2 with respect to the vertical axis, $x_4 = \dot{\theta}_2$ is the angle velocity of pole 2 with respect to the vertical axis, x_5 is the position of the cart, x_6 is the velocity of the cart, l_1 is the length of pole 1, l_2 is the length of pole 2, g is the acceleration of gravity, m_1 is the mass of the ball at the top of pole 1, m_2 is the mass of the ball at the top of pole 2, and m_c is the mass of the cart. Because of temperature, mechanical attritions, or others factors, the parameters of the system may change. As shown in Eq. (4), there will be Δf_i in the system equation. In order to check whether the proposed approach can cope with this situation, a constant C_u is introduced into the system function in our simulation. Thus, f_i in Eq. (26) becomes $C_u f_i$ for $i = 1, 2$ and 3 (or compared to Eq. (5), $C_u f_i = f_i + \Delta f_i$).

The parameters used in our simulation are: for the hitting (robust) control, $k_c = 15$, for the sliding surfaces of those subsystems; $c_1 = 5$, $c_2 = 3$, and $c_3 = 0.5$; for the hierarchical sliding surface, $k_{d2} = 0.05$ and $k_{d3} = 0.5$, and for the fuzzy compensator, the learning constant $\alpha = 300$. The initial states are $x_1(0) = 30^\circ$, $x_2(0) = 0$, $x_3(0) = -30^\circ$, $x_4(0) = 0$, $x_5(0) = 1m$, and $x_6(0) = 0$. Figures 2-4 show the response of all six states for $C_u = 1, 0.8$, and 0.4 , respectively. It can be observed the control performance is severely degraded when $C_u = 0.4$. Thus, in this study, the fuzzy compensator is implemented for the system with $C_u = 0.4$. The result is

presented in Figure 5. It can be found that the performance is significantly improved.

In order to have a clear comparison, the following indices are considered for those experiments.

- (i) Settling time: It is the time that the considered variable starts to stay in the range of $\pm\delta$. For each variable in a system, the δ is chosen independently.
- (ii) Average steady state error: It denoted the average error when the system enters the settling stage and for state x , it can be calculated as

$$\begin{aligned}
&\text{Average steady state error} \\
&= \frac{1}{b-a+1} \sum_{k=a}^b |x(k)| \quad (27)
\end{aligned}$$

where t is the total simulation time, $a = \frac{\text{settlingtime}}{0.01}$ and $b = \frac{t}{0.01}$.

The control performances in terms of those indices are tabulated in Tables 1 and 2. It is clearly evident that the proposed fuzzy compensator can have much better control performance.

In order to show the robust capability of the proposed approach, a noise as $\cos(2\pi t + 5) + 2 \cos(3\pi t + 1) + 5 \sin(2\pi t) + 2 \sin(3\pi t - 1) + 2 \sin(2\pi t - 2) + 3 \sin(\pi t - 3)$ is added into the system, where $C_u = 1$ is considered. The results are tabulated in Tables 3 and 4. Again, it is clearly evident that the proposed fuzzy compensator can have much better control performance.

6. Conclusions

A direct adaptive fuzzy sliding mode control has been proposed for under-actuated systems based on a hierarchical sliding control. Presently, the adaptive law design is not further modified to avoid the parameter drifting problem. But, it is known that the so-called robust modification techniques as discussed in [1, 21] can easily be applied to modify the proposed adaptive law for avoiding the parameter drifting problem. In our simulations, the doubled-inverted pendulum system is considered. From those simulations, it can be concluded that the proposed compensator can overcome those system uncertainties. Besides, it can be found that the compensator also provide good robustness properties.

Table 1. Settling times under various system uncertainties and with the use of the proposed fuzzy compensator

C_u variable	δ	1	0.8	0.4	0.4+fuzzy compensator
x_1	0.05	6.24	6.65	-	4.63
x_2	0.1	7.93	9.08	-	6.26
x_3	0.5	4.28	4.64	-	2.57
x_4	0.5	7.63	9.30	-	5.85
x_5	0.1	8.02	8.31	-	7.56
x_6	0.2	8.31	9.99	9.99	6.44
s_1	0.1	8.25	9.98	-	7.14
s_2	0.5	7.80	9.47	-	5.98
s_3	0.5	6.21	8.48	-	5.72
s_c	0.05	0.77	7.60	-	3.74

The notation (-) represents variable does not converge into the desired $\pm\delta$ region until the end of our simulation time.

Table 2. Average steady state errors under various system uncertainties and with the use of the proposed fuzzy compensator

C_u variable	1	0.8	0.4	0.4+fuzzy compensator
x_1	0.0139	0.0212	-	0.0115
x_2	0.0413	0.0587	-	0.0285
x_3	0.1197	0.1747	-	0.0986
x_4	0.2134	0.3111	-	0.1310
x_5	0.0348	0.0500	-	0.0411
x_6	0.1075	0.2945	0.2227	0.0728
s_1	0.0511	0.1297	-	0.0298
s_2	0.2393	0.3009	-	0.1420
s_3	0.1860	0.2566	-	0.0974
s_c	0.0028	0.0236	-	0.0133

The notation (-) represents variable does not converge into the desired $\pm\delta$ region and average steady state error is not defined.

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

References

- [1] R. Xu and U. Ozguner, "Sliding mode control of a class of underactuated system," *Automatica*, vol. 44, no. 1, pp. 233-241, 2008. <http://dx.doi.org/10.1016/j.automatica.2007.05.014>

Table 3. Settling times for systems with disturbance

Control input variable	δ	Without compensator	With fuzzy compensator
x_1	0.05	9.67	5.89
x_2	0.1	9.97	9.63
x_3	0.5	2.74	2.50
x_4	0.5	8.86	5.83
x_5	0.1	9.53	7.70
x_6	0.2	9.93	9.19
s_1	0.1	-	9.81
s_2	0.5	8.50	5.95
s_3	0.5	9.81	5.77
s_c	0.05	-	9.94

Table 4. Average steady state errors for systems with disturbance

Control input variable	Without compensator	With fuzzy compensator
x_1	0.0234	0.0131
x_2	0.0866	0.0474
x_3	0.1310	0.1092
x_4	0.1690	0.1907
x_5	0.0559	0.0426
x_6	0.1105	0.1086
s_1	-	0.0555
s_2	0.2518	0.2154
s_3	0.3192	0.1506
s_c	-	0.0483

Figure 1. The considered doubled-inverted pendulum system.

(a)

(b)

(c)

Figure 2. System responses without compensator for $C_u = 1$. (a) and (b) are the responses of all states, (c) the sliding surface s_c .

(a)

(b)

(c)

Figure 3. System responses without compensator for $C_u = 0.8$. (a) and (b) are the responses of all states, (c) the sliding surface s_c .

- [2] R. Olfati-Saber, "Normal forms for underactuated mechanical systems with symmetry," *IEEE Transactions on Automatic Control*, vol. 47, no. 2, pp. 305-308, 2002. <http://dx.doi.org/10.1109/9.983365>
- [3] X. Xin and M. Kaneda, "Swing-up control for a 3-DOF gymnastic robot with passive first joint: design and analysis," *IEEE Transactions on Robotics*, vol. 23, no. 6, pp. 1277-1285, 2007. <http://dx.doi.org/10.1109/tro.2007.909805>
- [4] R. Fierro, F. L. Lewis, and A. Lowe, "Hybrid control for a class of underactuated mechanical systems," *IEEE*

Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans, vol. 29, no. 6, pp. 649-654, 1999. <http://dx.doi.org/10.1109/3468.798069>

- [5] F. Yorgancioglu and H. Komurcugil, "Decoupled sliding-mode controller based on time-varying sliding surfaces for fourth-order systems," *Expert Systems with Applications*, vol. 37, no. 10, pp. 6764-6774, 2010. <http://dx.doi.org/10.1016/j.eswa.2010.03.049>
- [6] L. W. Lee and I. H. Li, "A Fourier series-based adaptive sliding-mode controller for nonlinear pneumatic servo systems," in *Proceedings of 2010 IEEE International*

(a)

(b)

(c)

(d)

Figure 4. System responses without compensator for $C_u = 0.4$. (a) and (b) are the responses for all states, (c) the sliding surfaces s_1 , s_2 , s_3 and s_c , (d) the control input u .

(a)

(b)

(c)

(d)

Figure 5. System responses with compensator for $C_u = 0.4$. (a) and (b) are the responses for all states, (c) the sliding surfaces s_1 , s_2 , s_3 and s_c , (d) the control input u and the compensator input.

- Conference on Fuzzy Systems*, Barcelona, 2010, pp. 1-5. <http://dx.doi.org/10.1109/FUZZY.2010.5584533>
- [7] D. W. Kim, H. W. Yang, J. S. Yoon, and H. P. Kim, "Decoupled adaptive fuzzy sliding-mode control," in *Proceedings of 2001 IEEE International Symposium on Computational Intelligence in Robotics and Automation*, Banff, Canada, 2001, pp. 96-101. <http://dx.doi.org/10.1109/CIRA.2001.1013179>
- [8] Y. J. Mon and C. M. Lin, "Hierarchical fuzzy sliding-mode control," in *Proceedings of 2002 IEEE International Conference on Fuzzy Systems*, Honolulu, HI, 2002, pp. 656-661. <http://dx.doi.org/10.1109/FUZZ.2002.1005070>
- [9] D. Qian, J. Yi, and D. Zhao, "Robust control using sliding mode for a class of under-actuated systems with mismatched uncertainties," in *Proceedings of 2007 IEEE International Conference on Robotics and Automation*, Roma, Italy, 2007, pp. 1449-1454. <http://dx.doi.org/10.1109/ROBOT.2007.363188>
- [10] C. M. Lin and Y. J. Mon, "Decoupling control by hierarchical fuzzy sliding-mode controller," *IEEE Transactions on Control Systems Technology*, vol. 13, no. 4, pp. 593-598, 2005. <http://dx.doi.org/10.1109/TCST.2004.843130>
- [11] S. Y. Shin, J. Y. Lee, M. Sugisaka, and J. J. Lee, "Decoupled fuzzy adaptive sliding mode control for under-actuated systems with mismatched uncertainties," in *Proceedings of 2010 IEEE International Conference on Information and Automation*, Harbin, China, 2010, pp. 599-604. <http://dx.doi.org/10.1109/ICINFA.2010.5512406>
- [12] H. F. Ho, Y. K. Wong, and A. B. Rad, "Adaptive fuzzy sliding mode control with chattering elimination for nonlinear SISO systems," *Simulation Modelling Practice and Theory*, vol. 17, no. 7, pp. 1199-1210, 2009. <http://dx.doi.org/10.1016/j.simpat.2009.04.004>
- [13] J. C. Lo and Y. H. Kuo, "Decoupled fuzzy sliding-mode control," *IEEE Transactions on Fuzzy Systems*, vol. 6, no. 3, pp. 426-435, 1998. <http://dx.doi.org/10.1109/91.705510>
- [14] L. Marton, A. S. Hodel, B. Lantos, and J. Y. Hung, "Underactuated robot control: comparing LQR, subspace stabilization, and combined error metric approaches," *IEEE Transactions on Industrial Electronics*, vol. 55, no. 10, pp. 3724-3730, 2008. <http://dx.doi.org/10.1109/TIE.2008.923285>
- [15] M. S. Park, D. Chwa, and S. K. Hong, "Decoupling control of a class of underactuated mechanical systems based on sliding mode control," in *Proceedings of SICE-ICASE International Joint Conference*, Busan, Korea, 2006, pp. 806-810. <http://dx.doi.org/10.1109/SICE.2006.315338>
- [16] L. C. Hung and H. Y. Chung, "Decoupled sliding-mode with fuzzy-neural network controller for nonlinear systems," *International Journal of Approximate Reasoning*, vol. 46, no. 1, pp. 74-97, 2007. <http://dx.doi.org/10.1016/j.ijar.2006.08.002>
- [17] L. X. Wang, *A Course in Fuzzy Systems and Control*. Upper Saddle River, NJ: Prentice-Hall, 1997.
- [18] J. L. Castro, "Fuzzy logic controllers are universal approximators," *IEEE Transactions on Systems, Man and Cybernetics*, vol. 25, no. 4, pp. 629-635, 1995. <http://dx.doi.org/10.1109/21.370193>
- [19] J. A. Farrell and M. M. Polycarpou, *Adaptive Approximation Based Control: Unifying Neural, Fuzzy and Traditional Adaptive Approximation Approaches*. Hoboken, NJ: Wiley, 2006.
- [20] L. X. Wang, "Stable adaptive fuzzy control of nonlinear systems," *IEEE Transactions on Fuzzy Systems*, vol. 1, no. 2, pp. 146-155, 1993. <http://dx.doi.org/10.1109/91.227383>
- [21] Y. C. Hsueh, S. F. Su, C. W. Tao, and C. C. Hsiao, "Robust L2-gain compensative control for direct-adaptive fuzzy-control-system design," *IEEE Transactions on Fuzzy Systems*, vol. 18, no. 4, pp. 661-673, 2010. <http://dx.doi.org/10.1109/tfuzz.2010.2045761>
- [22] Y. C. Hsueh and S. F. Su, "Learning error feedback design of direct adaptive fuzzy control systems," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 3, pp. 536-545, 2011. <http://dx.doi.org/10.1109/TFUZZ.2011.2178854>
- [23] J. Cao, X. Ji, P. Li, and H. Liu, "Design of adaptive interval type-2 fuzzy control system and its stability analysis," *International Journal of Fuzzy Systems*, vol. 13, no. 4, pp. 334-343, 2011.
- [24] C. H. Lee and H. Y. Hsueh, "Observer-based adaptive control for a class of nonlinear non-affine systems using recurrent-type fuzzy logic systems," *International Journal of Fuzzy Systems*, vol. 15, no. 1, pp. 55-65, 2013.
- [25] Y. C. Hsueh and S. Shun-Feng, "Direct adaptive fuzzy decoupled sliding control for a class of under-actuated uncertain systems," in *Proceedings of 2011 International Conference on Machine Learning and Cybernetics*, Guilin, China, 2011, pp. 226-231. <http://dx.doi.org/10.1109/ICMLC.2011.6016703>

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